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# The Mathematics Teacher

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*Mathematicians and automata*

PAUL BROCK

*Teaching arithmetic in the junior high school*

FOSTER E. GROSSNICKLE

*The Ganita-Sāra-Sangraha of Mahāvīracārya*

S. BALAKRISHNA AIYAR

*The official journal of*

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# Mathematicians and automata<sup>1</sup>

PAUL BROCK, *Chief, Mathematics Section, Computer Division,  
Consolidated Engineering Corporation, Pasadena, California.*

*A paper about one of the most modern developments in the field  
of mathematics—the computing field. Valuable information  
which all teachers need to adequately assist  
young people in selecting a suitable vocation.*

A DOZEN years have passed since Thornton C. Fry,<sup>2</sup> in his monograph on industrial mathematics, published under the aegis of the Bell Telephone Laboratories, made the following statements: "It is perhaps not too wide of the mark to estimate the total number (of mathematicians employed in industry today) at 150, not including actuaries and statisticians." He then continues, "Based on these (current estimates), a demand for new (mathematical) personnel of the order of ten a year may be predicted."

Dr. Fry did not anticipate the current situation with respect to employment of mathematicians. Today, graduate mathematicians and mathematicians with advanced degrees are at a premium in the labor market. The tremendous technological strides made during the past decade in electronics, nuclear energy and fluid dynamics, to name a few fields, have been accomplished to a large extent through mathematical analysis, and the advances have created a wealth of mathematical work, ranging from a tremendous volume of routine calculations to the most complex theoretical investigations. Mathematically trained people are critically needed to handle this work.

The schools where mathematicians are trained have been very slow to recognize this need. Hence, we are now in the almost impossible situation where very few mathematicians are being graduated by the schools yearly to meet this tremendous requirement. Perhaps it is characteristic of academic life to resist changes in society until convinced that a trend is not a transient fad.

A basic problem the schools must overcome is one of orientation at a secondary level. As a subject for study, mathematics has always had two stigmas attached to it. It has been a difficult subject and financially unrewarding. These stigmas have permeated the homes of secondary-level students, hence parental advice will quite often serve to steer students away from the study of mathematics. It will require a major effort on the part of school authorities to counteract these fears and firmly establish the fact that in our current society mathematics has become a vital, exceptionally interesting and particularly remunerative field of endeavor.

The schools must then orient the thinking of students in mathematics to the directions in which pupils of varying abilities and interests can use their talents.

I should like to discuss how mathematics is used in one particular field: the field of large-scale electronic calculating equipment. I shall try to indicate what

<sup>1</sup> Presented at the annual meeting of the National Council of Teachers of Mathematics, University of California at Los Angeles, December 29, 1953.

<sup>2</sup> T. C. Fry, *Industrial Mathematics* (New York: Bell Telephone Company, 1941).

type of curriculum at a college would be best suited to students who wish to enter this field as a vocation. But first, a few comments on the field itself.

#### THE COMPUTING FIELD

A large-scale computer is a piece of capital goods. From an individual's point of view, its cost is prohibitive. Some very small computer installations may be obtained for as little as \$10,000. The more powerful installations range well over \$1,000,000.

What do these machines do that a desk calculator can not do, to warrant the cost?

To compare a general-purpose digital machine with a desk calculator is to compare a Cadillac with a perambulator. Both vehicles have the capacity of conveying a person from Los Angeles to Chicago. In this sense both computers can do the same job. But let us make some comparisons.

	<i>Desk calculator</i>	<i>Computer</i>
Time for one addition	1 second	$10^{-2}$ – $10^{-5}$ seconds
Memory	2–3 numbers	500–500,000 numbers
Fully automatic	No	Yes
Cost	\$800	\$50,000–\$1,000,000
Technical servants	1.1	25–80
Mathematicians above the B.A. needed	0	15–70

The combination of speed, retentiveness, and the particular ability to respond to its own operational and logical requests in proper sequence make possible the solution of problems previously considered impossible from a practical point of view.

Several years ago I assisted with the solution of a system of fourteen first-order, non-linear ordinary differential equations, with thirty-two auxiliary defining algebraic equations. A third-order Milne scheme was selected to solve the system at an interval width of .02 second, for a 60-second range.

The automatic ability to obey sequentially an initially formulated program

allowed the machine to solve the problem at any time point, then correctly alter the solution values and continue to the new solution for the next time point, continuously through the complete solution. At all times the computer had to retain approximately three hundred partial results, hence the need for large memory. The over-all problem required approximately three million calculations.

This problem was run on the IBM Selective Sequence Electronic Calculator. Five years ago this machine was the only commercial operable digital computer in the country. Today it is defunct because it is too slow and too expensive to operate. I quote from a report<sup>3</sup> that was written on this problem, "The problem consumed 24.13 machine hours for solution on the SSEC; complete personnel time was approximately 6500 man-hours." If this ratio seems to be somewhat lopsided, it was estimated that hand calculation of the same work using desk calculators would have taken sixty man-years.

This problem was done in the so-called early days of large-scale computing. Today the labor would be reduced and so would the machine time on the most modern equipment. In fact, the same problem was recently run at the new RAYDAC installation at the Pt. Mugu, California, Naval Air Missile Test Center in one hour and four minutes.<sup>4</sup>

This is one illustration. In an operating computer laboratory one may be called upon to solve anything from a non-linear partial differential equation with complex boundary values to the problem of distributing a periodical magazine subscription for a large publishing house while maintaining a daily record of address changes, subscriptions, and a complete billing system.

<sup>3</sup> P. Brock and F. J. Murray, *Planning and error analysis for the numerical solution of a test system of differential equations on the IBM sequence calculator* (New York: Reeves Instrument Corporation, 1950).

<sup>4</sup> F. J. Murray, *Description of Hurricane computer acceptance tests* (Special Devices Center, U. S. Navy report, 1953).

#### THE MATHEMATICIAN'S ROLE

The high-school girl, punching a desk computer according to a preassigned plan will not be remotely aware that she is actually performing the iterative procedures of a relaxation scheme to determine a potential field within a cyclotron for a physicist at Oak Ridge. She is following detailed instructions.

In the same sense a machine can follow instructions, but people with technical ability must define the problem and propose a method of solution. They then must break the problem into a series of detailed instructions that the computer can recognize and obey.

Mathematical positions in the computer field may be divided into the following categories: analysts, programmers, coders, operators, administrators, designers.

1. *Analysts.* The analyst generally has responsibility for the problems that are presented to the computer installation for solution. He must fully understand the problem and maintain technical liaison with the proposer all during the solution process. In many cases he must determine the nature of the solution. Then he must set up a numerical procedure that will result in a reliable answer. He must be able to evaluate the result, giving estimates of the mathematical errors involved. These errors are the results of truncation, round-off and many other sources. Finally, he must put the results in a form understandable to the proposer and, in some cases, make suggestions assisting the proposer in decisions that are based upon the solution.

2. *Programmers.* The analyst decides on the numerical method; the programmer breaks down the numerical method into detailed operational steps that the computer can follow. This process is analogous to setting up a work sheet form for a human computer. On such a form the sequence of operations is established and itemized. This must also be done for the computer, but generally in a much more detailed fashion.

3. *Coders.* One additional operation that must be performed for computers, which is not required for hand computation, is the translation of the detailed operations into a code that the machine understands. Machines operate only on numbers. Thus, if the programmer stipulates an addition, the coder must translate this into a number, say 74, which may be the code for the operation. This code must then be placed upon an input medium. This may be Teletype or Flexowriter tape, magnetic tape, punched cards or in some cases may be typed directly into the machine.

4. *Operators.* In some installations, personnel are assigned the particular job of operating the computer. This work generally does not require a mathematical background. In fact, the operator will more often have engineering training so that he is in a position to determine the cause of machine troubles that occur during operations.

5. *Administrators.* It is necessary to have mathematical administrators at computer installations to coordinate the activities of the large numbers of mathematicians that are involved.

6. *Designers.* The designing of computing equipment is a fascinating subject that will not be discussed here. However, the subject is basically mathematical in character and mathematicians are playing a large role in this phase of the computing field.

The speed of large-scale computers is such that it is not unusual to spend two hundred man-hours preparing a problem for the machine, then to have the solution ground out in less than one hour. Keeping a machine busy therefore requires a large mathematical staff. It is not uncommon in machine laboratories to employ from fifteen to seventy mathematicians.

It is roughly estimated that by the end of next year there will be over fifty large-scale computing organizations in the country. With these figures in mind I should now like to discuss what I believe is desirable preparation to enter this field.

#### DESIRABLE PREPARATION

At the present time and for some time to come, the basic requirements for people entering the computing field will be a baccalaureate degree in mathematics. This is true of all the positions I mentioned, with the possible exception of machine operators. Senior positions, of course, require more academic training and experience.

It is a little difficult to describe a person who makes a good coder or programmer. The problem of coding is much akin to cryptography. It presents the same challenge to an individual that mathematical puzzles do, and handling this type of work requires an appreciation of puzzle- or problem-solving techniques. In addition, the coder and programmer must have a feeling for the problem involved and for the method that is being used to solve the problem, so that he may make proper decisions if necessary and be able to converse intelligently with the senior mathematicians responsible for the problem solution. Above all, his work must be accurate. Coding presents one of the most error-prone fields of detail work. To solve a problem accurately, the machine requires a perfect code. Many machine-hours are spent at computer installations, diagnosing and correcting faults in coding.

The basic training of a prospective entrant into this field is a well-rounded program of undergraduate mathematics. This program should be as extensive and intensive as that given to any college student seriously contemplating graduate work in mathematics. This type of program is the only one I know of to instill a mathematical discrimination and maturity in the individual sufficient to insure that he will execute his job well and be in a position to advance in the field.

I stated that further academic training is necessary for advancement in the field. This is very true because the senior positions involve responsibility for mathematical work and also include analysis of mathematical problems of wide and diverse origins.

Let me illustrate. A mathematician at a computer installation might be called upon to solve, hence to analyze, a problem involving heat transfer in the boilers manufactured by a large boiler-maker installation. It is unlikely that the person assigned to this job will have had any acquaintance with heat flow in boilers. Hence, he will have to acquaint himself very rapidly with the field and, in particular, with the phases bearing on the particular problem, so that he can abstract the mathematics and understand the relation between its solution and the practical application. This involves research in a field that probably has a well-developed literature. But it entails, to a certain extent, original research on subjects about which it is difficult for this individual to obtain information. The ability to do research and to work out uninhibitedly, in an original fashion, theory that may be well known to other people so as to expedite the solution time of the problem, is, I believe, the same type of ability that is supposedly developed in an individual when he is obtaining his advanced degrees in mathematics.

Thus, training and maturity in mathematics is a basic requirement, and more training and more maturity is necessary for more advanced work in the field. This training is independent of the particular branch of mathematics that the individual may be interested in. There are certain courses that are of particular value to a person entering this field. They are few in number and should not interfere with the general mathematical education of the individual involved. I would now like to discuss these courses.

There are three courses that I believe are important as preparation for work in the computer field:

1. Advanced calculus
2. Ordinary differential equations
3. Numerical methods

The advanced calculus course, or perhaps a series of courses, should include fundamental concepts of continuity, ana-

lyticity, complex functions, partial differentiation, variational methods, etc. These topics are covered in many advanced calculus courses that are given in the colleges today.

The traditional course in ordinary differential equations is of little use to the student. At best, it gives a chronological approach to the subject and illustrates some of the less rewarding thoughts of a number of eminent mathematicians. The course given to undergraduates classifies differential equations into such types as homogeneous, Clairaut's form, etc. This classification was originally set up to assist in the quadrature solution of ordinary differential equations. Today this classification is meaningless. In fact, even if a quadrature solution can be obtained, it is of much less use than a direct numerical solution to the equation.

From both a functional and a theoretical point of view, it is much more important to classify differential equations as linear or non-linear and then to study the special cases of each type. Thus, I feel that an elementary differential equation course should contain a complete treatment of the linear equation with constant coefficient case. The superposition principle for general linear equations should be stressed very strongly as a special case of linear operator theory. Also, some intuitive notions should be given on the effects of non-linearities in differential equations. The course should develop a familiarization with such concepts as damping, frequency responses, resonance, periodicity and many other common phenomena that arise in the solution of ordinary differential equations. The Weierstrass existence theorem should be stated. The notion of the solution of an equation existing in the neighborhood of a point should be elaborated. The fact that this neighborhood is very small and that most differential equation theory applies only locally and becomes very difficult when expanded over a large range, should be known when a student completes the basic course. Finally,

mention should be made of the standard practical methods that are used to solve differential equations. These are not by quadrature.

The solution of ordinary differential equations is one of the most common types of problems that computers are used for today. A student who has completed a course that includes the topics I have just mentioned will have a good background to handle this type of work. This course leads directly into the subject matter of advanced differential equations, and is excellent background for many other more sophisticated courses, such as partial differential equations, linear operator theory, calculus of variations, etc.

A natural extension of the differential equations course is a course in numerical methods. This is a subject that has been neglected in college mathematics curricula throughout the country. It also suffers from an acute case of traditionalism, and at best has trained people to do hand computational work. The advent of computers and hence the ability to do large computational tasks has caused mathematicians to reinvestigate the subject. Many of the standard textbooks used should at least be supplemented by modern methods.

I do not want to go into extensive detail on the topics covered in a numerical methods course. There are many topics and I feel it is difficult to cover the subject adequately in a three-semester course.

One comment I would like to make, however, is on the universal tendency to illustrate the ideas of numerical methods by considering simple illustrations. In many cases the heart of the computational problem cannot be reached unless the illustrative examples are large. One cannot effectively illustrate the bad effects of an ill-conditioned matrix by solving a two-by-two or a three-by-three system. One should use at least an eight-by-eight system. Of course, hand computation on a system of this type is unreasonable if one expects to get any work accomplished in

the semester. But this leads me to my final comment: No numerical methods course should be taught without a large-scale computer available to do the computational work.

A machine also serves to show that the numerical methods used must be considered for a machine that performs routine calculations and has not the power of selectivity when difficult computational effects are encountered as the individual would have. Let me illustrate. If, in the midst of an extensive calculation, a subtraction is encountered that reduces significance materially, a machine that has been programmed to perform the calculation must accept this loss of significance even if the coder did not originally plan for the possibility. (This is generally the case, because in many problems, matrix inversion for example, it is almost impossible to determine the results of intermediary calculations.) The human com-

puter, of course, at any stage of the calculation, can change the order of operations or recompute with extended accuracy if he deems it advisable. In any case he has his own ability available at all points of the calculation and can make use of it. Thus, numerical methods for machines involve problems that are certainly not traditional.

#### SUMMARY

The field of automatic computation is a budding giant. It is critically short-handed today and its needs will increase over the foreseeable future. Secondary and college teachers today have the responsibility for supplying the personnel needed. This can be done only by the teachers themselves finding out the needs of the field, giving this information to the students and their parents through orientation measures, and, finally, by supplying proper training for the students who are interested.

---

#### Paradox

Not truth, nor certainty. These I forswore  
In my novitiate, as young men called  
To holy orders must abjure the world.  
"If . . . , then . . . ," this only I assert;  
And my successes are but pretty chains  
Linking twin doubts, for it is vain to ask  
If what I postulate be justified,  
Or what I prove possess the stamp of fact.

Yet bridges stand, and men no longer crawl  
In two dimensions. And such triumphs stem  
In no small measure from the power this game,  
Played with the thrice-attenuated shades  
Of things, has over their originals.  
How frail the wand, but how profound the spell!

Clarence R. Wyllie, Jr.

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# Teaching arithmetic in the junior high school

FOSTER E. GROSSNICKLE, *State Teachers College, Jersey City, New Jersey.* The author points out that the acceptance of a theory of learning which emphasizes meaning and changing promotion policies in the elementary school must have an effect on the curriculum in the junior high school. The author discusses the resulting changes which, he feels, should be made in the junior high school.

## FACTORS INFLUENCING ARITHMETIC INSTRUCTION

THERE ARE THREE significant factors, which have a pronounced effect on arithmetic as it is taught today. The effect of these factors is very noticeable at the junior high school level. They are:

1. The acceptance of a theory of learning which emphasizes meaning and understanding

2. The acceptance of a plan of continuous promotion which is based on time spent in a grade rather than on achievement in various subject areas

3. The gradual deferment of topics to higher grades, rather than the traditional placement of these topics as planned two or more decades ago.

These factors may be classified as *psychological, administrative, and curricular*, respectively. It is commonplace to assert that a student should understand what he learns. Few teachers would challenge this statement. Whether to conform to the principle of meaningful learning does not provoke as great a diversity of opinion as the problem of what constitutes meaningful learning.

Within the past decade there has been a marked acceptance of the psychology of

learning which is based on meaning and understanding. This has been due to two factors. First, the results achieved predominantly from rote learning in arithmetic have not been satisfactory. Second, there is a growing body of scientific evidence which substantiates the worth of making a subject meaningful to the learner. The learner must be conscious of the structure of our number system and must know how the basic processes of arithmetic operate within the framework of the number system.

The second factor affecting the teaching of arithmetic is the growing acceptance of a policy of continuous promotion. According to this policy, a student's promotion is not contingent upon his achievement in a subject. The operation of a plan of continuous promotion is controversial. Shane expressed a pertinent view in regard to this plan. "The decision as to whether a child is to progress at the same rate as his age mate should be made only after careful study of the child's total development."<sup>1</sup>

The worth of a program of continuous promotion need not be appraised. The plan may be wrong in theory, but its wide

<sup>1</sup> Harold G. Shane, "The Promotion Policy Dilemma," *NEA Journal*, 42: 412 (October 1953).

acceptance in practice has created many new problems in classroom management for the teacher of arithmetic, especially in the upper grades.

The third factor which has affected the teaching of arithmetic has been the tendency to defer to higher grades the teaching of topics that formerly were included in the lower grades. As an illustration, several decades ago division with a two-place divisor was usually taught in the fourth grade. Most modern courses of study and textbooks in arithmetic introduce division with a two-place divisor in grade 5, the systematic treatment of fractions in grades 5 and 6, decimals in grade 6, and per cent in grade 7. About two decades ago all of these topics were taught at least one grade below the present level.

The Fifteenth Yearbook of The National Council of Teachers of Mathematics proposed two programs for use at the beginning of the ninth grade. One program recommended general mathematics and the other recommended the traditional college preparatory mathematics. A striking characteristic of each program is the assumption that a student finishing the eighth grade in a program which deferred teaching some topics to higher grades will have acquired the same amount of subject matter as the student who has completed the traditional program in the elementary school. There are two possible results from deferring topics through grades 3 to 6. If the traditional work of grades 7 and 8 is to be completed in the elementary school at the end of the eighth grade, either the subject-matter load of these two grades will be extremely heavy or some of the traditional work of these grades should be deferred to the ninth grade.

Having identified the factors affecting the teaching of arithmetic, it is in order to prescribe the necessary steps to be taken for solving the problems created by these forces. Certainly, there can be no unanimity of opinion pertaining to the solution of these problems.

#### HOW TO DEAL WITH FACTORS AFFECTING ARITHMETIC

The first and most important problem growing out of the factors affecting the teaching of arithmetic pertains to the interpretation given to a program which emphasizes meaning. There are many characteristics, or signposts, of a program which stresses meaning and understanding, but we shall be concerned with two guiding principles the teacher should consider. First, the teacher should have the student use materials which will enable him to discover relationships among quantities. Second, the teacher should understand the importance of growth in dealing with different levels of abstraction as a pupil learns a given process.

The terms *discovery* and *growth* are descriptive of the characteristics of a program which emphasizes meaning and understanding. The type of material, and how a student uses it, determines to a large extent the number of discoveries of relationships among numbers which he is able to make and the level of abstraction at which he is able to deal with quantities. The classroom should be a laboratory in which the student uses materials to discover facts and principles. In order to have a laboratory function properly, it must be equipped with necessary materials. The materials of arithmetic may be classified as *objective*, *visual*, and *symbolic*.

Objective or exploratory materials can be moved or touched, such as an abacus or a flannel board. Charts, posters, and pictures represent visual materials. The textbook and workbook are representative of symbolic materials.

The use of each of the three kinds of materials can be shown by dividing 3.2 by 2. Each student should possess a kit containing markers to represent any one-place whole number and markers to represent at least 20 tenths. To represent 3.2, he selects from his kit 3 markers to represent 3 ones and 2 markers to represent 2 tenths. He discovers that he cannot divide 3 ones into two equal groups as ones,

therefore he regroupes 1 of the ones as 10 tenths. The regrouped number is 2 ones and 12 tenths. Then he divides the number into two equal parts; each part is 1 one and 6 tenths.

At the next stage in the development, the student should study a visual presentation of the process. If the textbook does not give a visual representation, the teacher should make a diagram on the chalkboard and have the class explain each step in the process. Finally, the student should study the symbolic representation as given in his textbook. At this stage he should be able to explain each step in the algorithm.

The second principle which the teacher should recognize as a characteristic of meaningful learning is the sequence of steps for attainment of mastery in dealing with quantities. This principle is closely related to the use of materials in a developmental program. Meaningful learning is in evidence when a student's pattern of thinking progresses from an immature level of abstraction to successively higher levels of abstraction until he attains an adult level of mastery. When a student uses manipulative or visual materials in dealing with quantities, he operates at a low level of abstraction. When he works intelligently with symbols, without the use of any other learning aids, he operates at a high level of abstraction.

Good teaching is in evidence when a student is challenged to work at the highest level of abstraction at which he understands the work. Conversely, poor teaching is in evidence when a student operates at a low level of abstraction when he should be operating at a higher level. It is just as ineffective for a student to be working with objective materials when he should be working with symbolic materials as it is for him to be working with meaningless symbols when he should be dealing with exploratory or visual materials. The teacher should be able to determine the highest level of abstraction, and the kind of material to use at that

level, so as to challenge the student's ability to deal intelligently with quantities. The answer to this problem may be found from the student's response to questions which test his understanding of a given process. A study of a student's work habits gives valuable information about his understanding of a process. This information should be supplemented by a careful analysis of his thought pattern as he gives an oral solution to a problem or example.

The second problem enumerated in the introductory paragraphs pertains to the method of dealing with a group of students of varied abilities. Individual differences are accentuated at each succeeding grade when achievement is not the basis of promotion. The modal score on an achievement test in arithmetic made by a group of students at any given age group will be the representative score of that age group in a program of continuous promotion as well as in a program of promotion based on achievement. On the other hand, the range of the scores will be greater in a program of the first type than in a program of the other type. Under a plan of continuous promotion it is not unusual to find the range of achievement at the eighth-grade level to vary from the norms for grades 4 to 9. A wide range in achievement of this kind challenges the teacher to offer adequate provision for individual differences. The teacher must accept the philosophy that each student learns at his own rate and that this rate depends upon his background.

In a program of continuous promotion, the designation of grade level is only an administrative device for dealing with groups that are approximately homogeneous in chronological age. Readiness for new work in arithmetic depends primarily upon the background of the learner. Therefore, students in the upper grades who have wide ranges in experiences and backgrounds should not deal with the same subject matter in an identical manner.

Provision for individual differences in rates of learning may be achieved by considering four different factors. They are:

1. Variation in time
2. Variation in size of groups
3. Variation in subject matter
4. Variation in materials of instruction.

A policy of continuous promotion nullifies the factor of time in learning the subject matter of a given grade. The teacher should divide the class into groups in arithmetic in the same way that she forms different groups in reading. There are times when no grouping within the class is desirable. At the beginning of a new topic, such as per cent, the class should function as a whole. After the subject has been introduced, groups should be formed. When the results of diagnostic tests show specific weaknesses, those students who need help with the process should be formed into groups.

The formation of two or three groups which are approximately homogeneous is only an administrative procedure for meeting the problem of individual differences. Groups should be formed in order that the third factor, variation in subject matter, and the fourth factor, variation in the use of instructional materials, may become effective.

The differentiation of the curriculum offers one possibility for differences in ability among groups in a given class. The content for the slow learners should meet the rigorous criterion of social utility. On the other hand, the fast learners should have opportunities for dealing with materials which will enrich their mathematical experiences.

The most important factor affecting the problem of providing for individual differences within a class is the use of instructional materials. The group of fast learners in arithmetic needs much less work with objective and visual materials than the other groups. The slow learners need to operate with concrete materials for a longer period than the fast learners in order to acquire a background which

will enable them to deal intelligently with symbolic materials, such as given in a textbook. Therefore, the fast learners should be challenged to achieve much more than the minimum requirement of the course and should deal predominantly with symbolic materials. This group of students should make more insightful discoveries of number relationships than the slow learners. The slow learners should achieve mastery of that material which is socially significant by using objective and visual aids so as to discover the procedures to use in dealing with symbols. The extent that grouping enables a teacher to use suitable materials so that a student will be able to operate at his highest level of understanding determines the effectiveness of grouping as a means of providing for individual differences.

The third factor affecting the teaching of arithmetic in the junior high school results from deferring topics to later grades. Two changes, or modifications, of the curriculum should be made to adjust to this deferment of topics. First, a process should be spaced through several grades and not taught at one specific grade level. Second, the program in arithmetic should be lengthened so that it will not terminate at the eighth grade.

A process, such as division, should not be allocated to any one grade, but instead, it should be spaced through several grades. The inherent difficulty of the process should be an element in the grade placement of a topic. To illustrate, one of the most difficult examples possible in division with a two-place divisor is the example,  $16\overline{)9123}$ . This example should be taught at a much later development in the pupil's work in the subject than the example,  $21\overline{)672}$ . There is experimental evidence to show that an example in which an estimated quotient must be corrected to find the true quotient is more difficult for the pupil than an example in which the estimated quotient is the true quotient. Examples of the easier type should be taught approximately a year in

advance of the more difficult type. Division involving three-place divisors should be deferred until the seventh grade. This form of division is difficult and it has limited social usage. Therefore, the topic of division should be spaced through several grades, probably through three grades.

The second result of the program of deferment of topics is its effect on the work of the junior high school. Some of the topics which traditionally have been taught in the eighth grade should be deferred until the ninth grade. The course in mathematics for most students in this grade should be predominantly arithmetic. Those students who are preparing for engineering careers should be in a different curriculum. Such topics as finding the area and volume of unfamiliar figures should be deferred to the ninth grade so that the student then would be able to find the area, volume, or missing dimensions of these figures by the use of formulas and equations. The topic of per cent should be retaught at this grade level. The student should integrate the uses of per cent by discovering how each usage can be found by applying the formula,  $p=br$ . If the program of arithmetic is extended through the ninth grade, the student prolongs his instruction in mathematics an extra year. Carpenter<sup>2</sup> showed that in California approximately 50 per cent of the students were graduated from high schools in Los Angeles without having had training in mathematics at this level. Therefore, the corrective measure to apply to the problem created by deferring arithmetic is to have a spiral or vertical development in the curriculum and to extend the program of arithmetic through the ninth grade.

The three factors affecting the teaching of arithmetic in the junior high school and the proposed solutions to the problems created by these factors are:

<sup>2</sup> Dale Carpenter, "Planning a Secondary-Mathematics Curriculum to Meet the Needs of All Students," *THE MATHEMATICS TEACHER*, 42: 45 (January 1949).

1. Most teachers of arithmetic subscribe to a philosophy of learning based on discovery instead of repetition. This means that the classroom must be a place in which the student uses appropriate materials for discovering relationships among quantities.

2. The practice of continuous promotion intensifies the problem of individual differences among the students of a class. As a result the teacher must adapt instructional materials to meet the wide range of abilities represented in the class.

3. The tendency to defer topics in arithmetic has so modified the curriculum that these topics should be spaced through several grades and the arithmetic program extended through the ninth grade. It is not to be inferred that the program should terminate there. The senior high school must offer a program in mathematics in which provision is made for maintaining basic skills in arithmetic.

#### GROWTH AND PROVISION FOR INDIVIDUAL DIFFERENCES

In light of the factors which affect the teaching of arithmetic in the junior high school, the teacher should provide for:

1. Growth in dealing with numbers
2. Differences in level of operation with number.

Many teachers in the junior high school do not provide for growth in the student's level of thinking in dealing with the basic operations. These teachers look upon the *product* of learning as the essential element in learning. Therefore, much of the work in the four basic processes dealing with integers and with common and decimal fractions is merely a review of these topics with emphasis on speed and accuracy of computation. One writer speaks of telescoping the review work and thus emphasizing the product of learning.

The *process* of learning is a vital factor because the process deals with the thought pattern of the learner. At the junior high school level the student should approximate an answer to see that it is sensible.

Ability to approximate an answer represents a much higher level of thinking with numbers than performing a given algorism and checking it to see that the computation is correct.

As the student performs the basic operations with integers, he should discover some of the essential principles which pertain to grouping with each of the four fundamental operations. The following generalizations or principles apply equally well to algebra or general number as they apply to arithmetic number. The generalizations<sup>3</sup> are:

A. Addition

1. Only like quantities can be added.
2. Quantities can be added in any order.

B. Subtraction

1. Only like quantities can be subtracted.
2. If either of two numbers is subtracted from their sum, the remainder is the other number.

C. Multiplication

1. The order in which two or more numbers are multiplied does not affect the product.
2. An indicated sum of two or more terms can be multiplied by a number providing each term of the indicated sum is multiplied by that number.
3. An indicated product of two or more numbers or factors can be multiplied by a number provided only one of the factors of that product is multiplied by that number.

D. Division

1. An indicated sum of two or more numbers can be divided by a number by dividing each term of the sum by the number.
2. Dividing by a number is the same as multiplying by the reciprocal

of that number. Similarly, multiplying by a number is the same as dividing by the reciprocal of that number.

3. An indicated product of two or more factors can be divided by a number by dividing only one of the factors by that number.
4. If the product of two numbers and one of the numbers are given, the missing number can be found by dividing the product by the given number.
5. The value of a fraction is not changed if both numerator and denominator are multiplied or divided by the same number, except zero.

These principles plus the principle which governs the order of sequence of different operations are the essential mathematical principles governing the basic operations with integers. The student at the junior high school level should understand how each principle applies in a given situation. These principles are not to be taught as a unit or body of subject matter. The teacher should understand each principle and should be aware of situations in which one or more of these principles apply. To illustrate, the student may evaluate the formula for the perimeter of a rectangle, as  $p=2(4+7)$ . The conventional way is to add 4 and 7 and then multiply 11 by 2. The teacher should have the student find the answer by multiplying each term of the indicated sum by 2 and then adding those products. Then he should generalize about the experience. The student learns the principle when he generalizes about his experience. The skillful teacher is always alert to quantitative situations in which the insightful student should discover the operation of a mathematical principle.

The principles enumerated pertain solely to the mathematical phase of number. Most of the slow learners in arithmetic will profit little from instruction aimed at

<sup>3</sup> The writer is indebted to Dr. John Reckzeh, State Teachers College, Jersey City, New Jersey, for help in formulating these principles.

an understanding of these principles. These generalizations dealing with the basic processes are intended primarily for superior students who develop insight into number. Therefore, the teacher in the junior high school should provide for growth in dealing with integers by having the student approximate an answer to see that it is sensible and to emphasize the basic principles which govern the operations with numbers. The degree of achievement in each of these two fields depends upon the background of the student and his growth in dealing with quantities.

The following illustration shows how to vary the method of procedure in dealing with common fractions among groups of different abilities. The teacher should not anticipate that the slow learners will develop the same insight into the structure of our number system that the superior students will develop. Mathematical principles which govern the operations with fractions should be related to corresponding principles which govern the operation with integers.

One of the basic concepts in the operation of fractions is the meaning of expressing fractions in lower or higher terms. All students should understand that reducing a fraction to lower terms means dividing a whole so as to make the size of the equal parts larger, but to make the number of parts smaller. The reverse is true in changing a fraction to higher terms. In this case it means to divide a whole so as to make the size of the equal parts smaller, but to make the number of equal parts larger. The teacher should use a fractional chart to enable a student to make these generalizations. The use of fractional cutouts also should enable him to discover the relationship between the number of parts into which a whole is divided and the size of these parts. Then each group of students should see that a fraction may be reduced by dividing both numerator and denominator by the same number without changing the value of the fraction. Similarly, both terms of a fraction may be multiplied

by the same number without changing the value of the fraction. By use of objective and visual materials, students in all groups at the junior high school level should understand the two basic generalizations stated. The superior students also should understand why it is possible to multiply or divide both terms of a fraction by the same number without changing the value of the fraction. He should discover that multiplying or dividing both terms of a fraction by the same number is the same as multiplying or dividing the fraction by 1. To illustrate,  $\frac{3}{4}$  may be multiplied by 1 in the form of  $\frac{4}{4}$  to give the fraction  $\frac{12}{16}$ . On the other hand, 1 added to a number or subtracted from a number gives an answer which is different from the given number. Therefore, the same number can not be added to or subtracted from both terms of a fraction without changing the value of the fraction. The student can verify the principle by adding 1 to or subtracting 1 from both terms of the fraction  $\frac{5}{6}$ . Few students in junior high school, or in higher grades, understand that the same number can not be added to or subtracted from both terms of a fraction without changing the value of the fraction. The reason is not due to the difficulty of the mathematical principle involved. Very probably the teacher never presented to the student the opportunity to discover the relationship between operations with integers and fractions.

Provision for differences in background in fractions should be made by differentiating the curriculum. For slow learners, the computational phase of fractions should be limited to those examples which are likely to be used in socially significant situations. Further provision for these differences should be made by the method used for presenting a topic in fractions. The slow learners must make greater use of objective materials than fast learners. The slow learners will be able to make few of the discoveries of mathematical principles governing the operations which

may be performed with fractions. On the whole, the slow learner should study fractions for their utilitarian value. The fast learner should not only be mindful of the social usefulness of fractions, but also be able to discover mathematical relationships among quantities and to understand the basic principles which govern the operations with fractions.

#### SUMMARY

It has been shown that the arithmetic program in the junior high school is affected by three factors, two of which are especially noteworthy. The first of these factors is the wide acceptance in principle, but not in practice, of a philosophy of learning which is based on meaning and understanding. This means that a student learns best when he discovers a procedure for himself. He should usually first learn a process at a low level of abstraction by using objective materials and progress through successively higher levels of abstraction until he is able to think intelligently with symbols without the use of supplementary aids. The second of these factors is the wide acceptance of continuous promotion which accentuates the problem of providing adequately for individual differences in the upper grades.

Provision for these differences may be effected chiefly by supplying the student with different kinds of materials with which to work. The teacher should understand that slow learners will not be able to discover many of the mathematical principles governing the operations with numbers and the structure of the number system that the superior students will discover.

Therefore, the teacher of arithmetic in the junior high school needs two types of materials to implement a program of the kind described. First, the teacher needs reliable tests in specific areas to test the student's level of understanding at which he is able to operate. And second, the teacher needs a variety of instructional materials which are geared to meet the needs of the students at their respective levels of operations. A classroom supplied with these two types of materials is equipped for an effective program for teaching arithmetic in the junior high school. Finally, and most important of all, the teacher must understand how to use these materials wisely. The teacher should stimulate the student to operate at the highest level of abstraction at which he is able to succeed and to understand the work.

#### A note on writing fractions

In algebra classes it sometimes happens that expressions involving fractions are misinterpreted because of the way in which the fractions are written. It is a common practice to use a slanting line to separate the numerator and denominator of a fraction as, for example,  $\frac{3}{8}$ . In single fractions of simple form this causes no confusion, but if a fraction is to be multiplied by some quantity or if some quantity is to be added to the fraction, the use of the slanting line might lead to misinterpretation. Thus  $3n/5(n+2)$  might be read either as  $\frac{3n}{5(n+2)}$  or as  $\frac{3n}{5}(n+2)$ .

Likewise  $8/7x+3$  might be read either as  $\frac{8}{7x}+3$  or as  $\frac{8}{7x}+3$ . Such ambiguities can be avoided by

using a horizontal line instead of a slanting line to separate the numerator and denominator of every fraction. It is recommended that students be taught to adhere to this practice consistently in all work which involves the writing of fractions.—Charles H. Butler, Western Michigan College, Kalamazoo, Michigan.

# The Ganita-Sāra-Sangraha of Mahāvīrācārya

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*An account of the imaginative approach to mathematics  
employed by early Hindu mathematicians as found in  
a well-known book by Mahāvira*

IN THE HISTORY of human civilization, two countries are prominent in learning and culture—Greece in the West and India in the East. It is to the latter that our attention is now to be directed. History records the great heights achieved by the Hindus in the realm of mathematics. Whatever might be the reason for the stagnation in the stretch of time between the end of the twelfth and the end of the nineteenth centuries, there is no gainsaying the fact that for better than a thousand years prior to this period of stagnation, India produced mathematicians of high rank and solid achievement in many of the branches of that science.

There is, however, considerable speculation and uncertainty regarding the exact dates and periods of early Hindu mathematical achievement. According to G. R. Kaye,<sup>1</sup> three periods can be broadly indicated within which the mathematical literature and activity can be spread:

- (1) The *Vedic and S'ulvasūtras period*, ending about 200 A.D.
- (2) The *astronomical period*, 400 to 600 A.D.
- (3) The *Hindu mathematical period proper*, 600 to 1200 A.D.

The chief centers of development in Northern India were Ujjain and Pāṭali-

putra (Patna). Down south, the tract of country in South India comprising Mysore and the neighboring districts was inhabited by people whose mother tongue was Kannada. About the middle of the third period referred to above, the Rāshha Kūta line of kings were ruling over the area. To the court of one of them, King Amōghavardana Vripatunga (814-877 A.D.), was attached a man of great learning, the astronomer and mathematician, Mahāvīrācārya. Chronologically, therefore, he belongs to the time between two of the other great Hindu mathematicians, Brahmagupta (c. 628) and Bhāskarā-Chārya (c. 1150), which on all accounts is regarded as a fairly accurate conclusion.

Mahāvira was a Jain by religion. The study of mathematics was very popular among the Jain scholars. Indeed it was accorded the status of one of the four *anuyogās*, which were the auxiliary sciences, the study of which helped the aspirant to the attainment of soul-liberation. Mahāvira's claim to fame rests on the firm foundation of his great work, the *Ganita-Sāra-Sangraha*. Its name indicates its technical nature, which is the usual characteristic of books which summarize knowledge of a subject up to the time of its production. The work seems to have been popular in his part of the country and much appreciated by the students and lovers of mathematics.

<sup>1</sup> G. R. Kaye, *Indian Mathematics* (Calcutta: Thacker, Spink and Co., 1915), p. 3.

It is to be noted, however, that, in spite of the technical nature of its content and development, and recognition of its practical helpfulness, it is valued more for its historic significance than as a mathematical treatise. A comprehensive and scholarly edition of the treatise was brought out by the late professor M. Rangācārya, Professor of Sanskrit of the Presidency College, Madras, with an accurate translation and elaborate notes.<sup>2</sup> It is included in the bibliography of books on the history of mathematics in THE MATHEMATICS TEACHER, with the comment that it is "the greatest contribution made in a century to the history of Hindu mathematics. A translation with notes of an extensive work of the ninth century."<sup>3</sup>

By and large, the *Ganita-Sāra-Sangraha* is regarded as the most scholarly treatise in Hindu mathematics. A casual comparison with the works of Brahmagupta and Bhāskara brings out a general similarity of topics with appreciable differences. These differences pertain to methods, rules of work and much of the illustrative material, while similarity is discernible in the general spirit that inspires and pervades their major topics and many problem situations.

The language is elliptical and riddle-like. As F. Cajori says, "The Indians were in the habit of putting into verse all mathematical results they obtained, and of clothing them in obscure and mystic language which, though well adapted to aid the memory of him who has already understood the subject, was often unintelligible to the uninitiated."<sup>4</sup>

The treatment is clothed in verses employing many of the popular metrical modes which, by their variety, relieve monotony and appeal to our aesthetic sense.

<sup>2</sup> M. Rangācārya. *The Ganita-Sāra-Sangraha of Mahāvīracārya* (Madras: Government Press, 1908).

<sup>3</sup> "A Brief List of Mathematical Books Suitable for Libraries in High Schools and Normal Schools," THE MATHEMATICS TEACHER, Vol. XVIII, p. 481.

<sup>4</sup> Florian Cajori, *A History of Mathematics* (New York: The Macmillan Company, 1919), p. 83.

Coming to the subject matter itself—Mahāvīracārya being a staunch Jain by religion, the opening invocation is addressed to Lord Jina. He is extolled as the shining lamp of the knowledge of numbers (*Sankyā-gnāna-pradīpa*), by which is illuminated (*prakāśitam*) the whole universe. In fact, stanzas in praise of Jina-deva occur at the beginning of almost all the sections of the treatise.

After the invocation follows an appreciation of the science of calculation (*ganita-sāstra-prasamsa*). There is no aspect of life and no phenomenon in this world and in all the worlds that is free from the control and application of the principles of mathematics. After a comprehensive enumeration of these aspects, the author concludes, "What is the good of saying much in vain? Whatever there is in all the three worlds which are possessed of moving and non-moving things—all that indeed cannot exist as apart from measurement."<sup>5</sup>

With an exuberance of poetic imagination characteristic of many Hindu authors, the divisions of the book find enumeration thus: *Ganita-Sāra-Sangraha* is a vast ocean. Terminology (*samgūya*) provides its waters, which stand impounded by the eight arithmetical operations (*parikarma*). Fractions and operations with them (*kalāsavarna*) are its innumerable rolling fish, and crocodiles abound in the shape of miscellaneous examples (*prakīrnaka*). Rule of three (*thri-rāsika*) lashes up its waters into waves, and mixed problems (*misraka*) are the gems which by their navigated splendor impart luster to the deep. Its extensive bed itself is formed of area problems (*kshetra-visthīrma*) with cubic measurements (*Khāta*) providing mounds upon mounds of sand. Shadow reckoning and connected references to astronomical matters (*chāya*) are the advancing tides. Into this vast ocean of the *Sangraha*<sup>6</sup> should the arithmeticians

<sup>5</sup> M. Rangācārya, *The Ganita-Sāra-Sangraha of Mahāvīracārya* (Madras: Government Press, 1912), p. 3.

<sup>6</sup> *Sangraha* means a synopsis of the essentials of mathematics.

dive to gather in abundance the pure gems of their desire.

Indeed, so riotous has been the author's imagination in some places that it oversteps the limits of discretion and introduces situations that call forth our amusement. Particularly in the applications of the principles and rules, there is a strange admixture of the significant and the fanciful. Here is a sample: "A well completely filled with water is ten *dandas* in depth; a lotus sprouting up therein grows from the bottom at the rate of  $2\frac{1}{2}$  *angulas* in a day and a half; the water flows out through a pump at  $2\frac{1}{2}$  *angulas* in  $1\frac{1}{2}$  days;  $1\frac{1}{2}$  *angulas* of water are lost in a day by evaporation owing to the rays of the sun; a tortoise below pulls down  $5\frac{1}{4}$  *angulas* of the stalk of the lotus plant in  $3\frac{1}{2}$  days. By what time will the lotus be on the same level with the water in the well?"<sup>7</sup> (N.B.: 6 *angulas* = 1 foot, 96 feet = 1 *danda*.)

In the caustic words of the Arab mathematician, Alberuni, there is to be found in Hindu mathematics and astronomy a mixture of "pearl shells and sour dates, or of pearls and cow dung, or of costly crystals and common pebbles."

Then Mahāvira steps aside to indicate who should be regarded as properly equipped for studying the science of mathematics. It is not to be easily undertaken by all and sundry. "By means of the eight qualities, viz., quick method of working, forethought as to whether a desirable result will be produced, or an undesirable result arrived at, freedom from dullness, correct comprehension, power of retention, and the devising of new means of working, along with getting at those numbers which made (unknown) quantities known—(by means of these qualities) an arithmetician is to be known as such."<sup>8</sup>

Then follow the enumeration and definition of units and the relations among them, tables of weights and measures as we call them. The terminology (*paribhāṣha*)

includes space (*kṣetra*), time (*kāla*), and other entities like quantities of grain, gold, silver, etc.

Mahāvira enumerates 1 to 9 and zero by name. In the ascending scale of integers, twenty-four rank-names are given, beginning with one (*ēka*) and mounting up to  $10^{23}$  (*mahākṣhōbhe*, a number formed by 1 followed by 23 zeros), each of which is thus ten times the preceding one.

Nominal numerals are also used. They are the names of objects and object-groups, suggesting the particular many-nesses of the corresponding numbers in view. For example, *moon* stands for one, *oceans* for four, *Rudras* for eleven and so on. Zero (*sūnya*, void) is indicated by names which are synonymous with the sky (*akāśe*, *gagara*). The use of such notation made it possible to represent a number in several ways. This greatly facilitated the framing of verses containing arithmetical rules and scientific constants, which could thus be more easily remembered.

In addition to the indicating of the digits by pure number names and nominal numerals exclusively, there is the additional device of combining the two modes in the same sentence, which is adopted obviously for the purpose of "securing metrical convenience and avoidance of cumbrous ways of stating numbers."

The decimal system is followed. The digits are set down in two ways, from left to right or right to left beginning with units, the order being determined by the student from the context largely.

The eight operations dealt with as fundamental are multiplication (*guṇa-kāra*), division (*bhāga-hāra*), squaring (*krīṭi*), square root (*vargamūla*), cubing (*ghana*), cube root (*ghana-mūla*), series (*chiththi* or *samkalita*) and subtraction of parts of series (*śeṣha* or *vyuthkalita*).

The rules of work (*karana sūtras*) regarding these are given, but the several steps are not detailed. In this sense, the name of the treatise is significant; it is a *sangraha*, a synopsis of the essentials of

<sup>7</sup> M. Rangācārya, *op. cit.*, p. 89.

<sup>8</sup> *Ibid.*, p. 8.

mathematics. Alternative methods and different applications are also provided. "Although the great Hindu mathematicians doubtless reasoned out most or all of their discoveries, yet they were not in the habit of preserving the proofs, so that the naked theorems and processes of operation are all that have come down to our time."<sup>9</sup>

The operations of addition and subtraction receive no separate attention for the apparent reason that they are obvious and simple.

Three alternative rules of multiplication are stated, two of which are obviously to facilitate working through choice of suitable factors. These rules are: "(1) After placing (the multiplicand and the multiplier one below the other) in the manner of the hinges of a door, the multiplicand should be multiplied by the multiplier, in accordance with (either of) the two methods of normal (or) reverse working, by adopting the process of (i) dividing the multiplicand and multiplying the multiplier by a factor of the multiplicand, (ii) of dividing the multiplier and multiplying the multiplicand by a factor of the multiplier, or (iii) of using them (in the multiplication) as they are (in themselves)."<sup>10</sup>

The product-patterns are often given fanciful names. Thus: "In this (problem), 12345679 multiplied by 9 is to be written down; this (product) has been declared by the holy preceptor Mahāvira to constitute the necklace of Narapala."<sup>11</sup>

The bare rules of division are given, thus: "Put down the dividend and divide it, in accordance with the process of removing common factors, by the divisor, which is placed below that (dividend), and then give out the resulting (quotient)" and, "The dividend should be divided in the reverse way (i.e., from left to right) by the divisor placed below, after performing in relation to (both of) them the operation

of removing the common factors, if that be possible."<sup>12</sup>

The division problem situations refer mainly to distribution of coins, gems, fruits, etc., among men and temples.

The next four sections on squaring, square root, cubing, and cube root are more or less alike in treatment. They begin with bare statements of rules of work, with five or six stanzas embodying exercises for application. Thus the rule for squaring numbers is: "The multiplication of two equal quantities: or the multiplication of the two quantities obtained (from the given quantity) by the subtraction (therefrom), and the addition (thereunto), of any chosen quantity, together with the addition of the square of that chosen quantity (to that product): or the sum of a series in arithmetical progression, of which 1 is the first term, 2 is the common difference, and the number of terms wherein is that (of which the square is) required: gives rise to the (required) square."<sup>13</sup>

Many of these rules, if set in modern algebraic notation, bear resemblance to formulas with which we are familiar. In modern notation the rules for squaring a number are: (i)  $a \times a = a^2$ ;

$$(ii) (a+x)(a-x) + x^2 = a^2$$

and

$$(iii) 1+3+5+7+\text{to } a \text{ terms} = a^2.$$

The next two sections on series give working rules for finding the sum, number of terms, common difference and first term of an arithmetical progression and the first term and common ratio of a geometrical progression. The exercises that follow are of a mechanical nature.

With regard to operations with zero, we gather that (1) a number remains unchanged when it is combined with or diminished by zero; (2) a number multiplied by zero is zero; (3) every number divided by zero is left unaltered. This last assertion is apparently based on the notion that "division by zero" means that there is no

<sup>9</sup> Cajori, *op. cit.*, p. 83.

<sup>10</sup> M. Rangācārya, *op. cit.*, p. 9.

<sup>11</sup> *Ibid.*, p. 10.

<sup>12</sup> *Ibid.*, p. 12.

<sup>13</sup> *Ibid.*, p. 13.

number to divide by, and the original number (dividend) is therefore left alone.

The rules for the product of negative numbers are also stated.

Fractions receive a relatively more elaborate treatment than integers. They are of six types: simple (*bhāga*), fraction of a fraction (*prabhāga*), complex fraction (*bhāgabhāgya*), associated fractions (*bhāgyabandha*), dissociated fractions<sup>14</sup> (*bhāgāpavāha*), and combinations of the above varieties (*bhāgamātra*).

The eight fundamental operations are equally applicable to integers and fractions. The statements of them are often longwinded and complicated. There is practically no attempt to obtain generalized procedures. Each type of fraction is treated on its own. The exercises involve mostly abstract numbers and include no recreational variety as in the case of integers.

The section on miscellaneous problems envisages ten important varieties. They relate to fanciful types and a good number of them are very complicated, so much so, that all the ingenuity of the reader is required to unravel and set down the data figures and the relations among them in a manageable analytical form to aid manipulation and solution, with the help of problem-solving techniques employing equations. Many problems involve squares and square roots of collections (numbers) of things, wholes or parts thereof and intermediate remainders, and solutions can without exaggeration be regarded as computational and manipulative gymnastics.

<sup>14</sup> Editor's Note: There are two kinds of associated fractions according to Mahāvīracārya. The first is a fraction associated with integer, i.e., a mixed number. The second consists of fractions associated with fractions, e.g.,  $\frac{1}{2}$  associated with  $\frac{1}{3}$  means  $\frac{1}{2} + \frac{1}{3}$  of  $\frac{1}{4}$ .

There are also two kinds of dissociated fractions. The first is the difference between an integer and a fraction. The second consists of fractions dissociated from fractions, e.g.,  $\frac{1}{2}$  dissociated from  $\frac{1}{3}$  means  $\frac{1}{2}$  of  $\frac{1}{3}$  is to be subtracted from  $\frac{1}{3}$ .

The rule of three<sup>15</sup> includes direct and inverse varieties of ratio relations and compound proportions involving single, double, treble and quadruple combinations. The problems given are often fanciful as is illustrated by the following: "A powerful unvanquished excellent black snake, which is 32 hastas in length, enters into a hole (at the rate of)  $7\frac{1}{2}$  angulas in  $\frac{5}{14}$  of a day; and in the course of  $\frac{1}{4}$  of a day its tail grows by  $2\frac{1}{4}$  of an angula. Oh ornament of arithmeticians, tell me by what time this same (serpent) enters fully into the hole."<sup>16</sup>

Very complicated applications of the rule of three, involving investments problems, barter and exchange of commodities are given. For example: "Twenty horses, (each) of 16 years (of age), are worth 100,000 gold coins. Oh leading arithmetician, say how much 70 horses, (each) of 10 years (of age), will be (worth) at this (rate)."<sup>17</sup>

Proportionate division of proceeds and profits are then dealt with and involve ratios with integral and fractional terms. They lend themselves to be treated with the help of simple, simultaneous, quadratic and radical equations. We also have a large collection of problems on calculations relating to transactions with gold.

Then there are problems to test logical argument and validity or otherwise of conclusions. Others refer to progressions, time and distance, relative velocity, time and work, and so on.

<sup>15</sup> Editor's Note: The rule of three is simply a mechanical device for solving proportions in which one of the four quantities is unknown. Thus a proportion, which in modern notation would be  $a:b::c:x$ , would be solved by writing  $a-b-c$  and the rule is to multiply the second and third numbers and divide by the first. In more complicated cases such as  $a:b:c::x:d:e$  the numbers were written



and the lines indicated that the product of  $a$ ,  $d$  and  $c$  were to be divided by the product of  $b$  and  $e$ . See Vera Sanford, *A Short History of Mathematics* (Boston: Houghton Mifflin Company, 1930), p. 162-163.

<sup>16</sup> M. Rangācārya, *op. cit.*, p. 90.

<sup>17</sup> *Ibid.*, p. 91.

The rules on areas in this chapter relate to the mensuration of triangles (equilateral, isosceles and scalene), quadrilaterals (squares, rectangles, trapeziums, etc.), circles and their parts, ellipses, conchoidal forms (*kambuka vritta*), areas of concave and convex spherical surfaces, annular areas, segments of circles, and other complex miscellaneous configurations like surfaces of grains (*yava*), drum (*muraja*), trumpet (*panava*), etc. Patterns built up of triangles, squares, and circles are offered for area finding.

In this connection it will be of interest to note the rule for finding the area and circumference of a circle. "The (measure of the) diameter multiplied by three is the measure of the circumference; and the number representing the square of half the diameter, if multiplied by three, gives the (resulting) area in this case of a complete circle."<sup>18</sup>

After dealing with relatively simple cases, the author passes on to the consideration of what he styles as devilishly difficult problems. They require the construction and use of involved formulas. As an example: "In the case of an equilateral quadrilateral figure, the (numerical) measure of the diagonal is equal to (that of) the area. What may be the measure of (its) base?"<sup>19</sup>

Excavations (volume) come next. The sides of pits and cavities illustrate a variety of geometrical shapes. Rules are stated for finding the cubic contents of solids bounded by spherical surfaces (*golākasa-kshetram*), cylinders, triangular pyramids, piles of bricks, walls of different shapes, broken walls, water flowing

through pipes, etc. Examples are appended.

We have already seen that Mahāvira was a notable astronomer, like the other great mathematicians of India. The section on shadows furnishes some mathematics for many astronomical problems. The rules relate, among other things, to finding directions relative to the due East, shadow reckoning with the gnomon (sundial) consisting of a circle with a style planted at the center, lengths of shadows near sunrise and sunset and at intermediate times, equatorial shadows (when sun passes through the First Points of Aries and Libra), shadows at places where there are no such shadows. Many problems remind us of the common types or heights and distances found in our textbooks. For example: "The shadow of a pillar, 12 hastas (in height), is 24 hastas in measure. At that time, Oh arithmetician, of what measure will the human shadow be?"<sup>20</sup>

This brings us to the end of a brief survey of an outstanding work on Hindu mathematics.

Professor D. E. Smith, American mathematician and historian of mathematics, in his valuable and scholarly introduction to Prof. Rangācārya's edition of the *Ganita-Sāra-Sangraha* says, "... oriental mathematics possesses a richness of imagination, an interest in problem solving and poetry, all of which are lacking in the Treatises of the west, although abounding in the works of China and Japan."<sup>21</sup> The Arabs were particularly attracted by the poetical, the rhetorical and the picturesque in the Hindu treatment of the subject, more than by the abstract approach.

<sup>18</sup> *Ibid.*, p. 189.

<sup>19</sup> *Ibid.*, p. 221.

<sup>20</sup> *Ibid.*, p. 278.

<sup>21</sup> *Ibid.*, p. 24.

## Instructions to authors of manuscripts

Manuscripts that come to the editorial office of THE MATHEMATICS TEACHER should be prepared according to the following directions. Much labor will be saved by both author and editor if the considerations, listed below, are kept in mind when preparing manuscripts for publication in THE MATHEMATICS TEACHER.

1. All manuscripts and editorial correspondence should be sent to:

H. Van Engen, Editor  
THE MATHEMATICS TEACHER  
Iowa State Teachers College  
Cedar Falls, Iowa

2. All manuscripts should be typewritten, double or triple spaced, on 8½"×11" good quality paper. The manuscripts should be typed so as to leave one-inch margins on both sides (one and one-half inches is even better).

3. References and footnotes should be numbered consecutively and placed at the bottom of the page. Prepare footnote references as follows:

To a book:

<sup>1</sup> Francis G. Lankford, Jr. and John R. Clark, *Basic Ideas of Mathematics* (New York: World Book Company, 1953), p. 115.

To an article in a periodical:

<sup>2</sup> Jack D. Wilson, "Arithmetic for Majors?" THE MATHEMATICS TEACHER, Vol. XLVI (December 1953), p. 260.

To a yearbook:

<sup>3</sup> Maurice L. Hartung, "Motivation for Education in Mathematics," *Twenty-first Yearbook*, The National Council of Teachers of Mathematics, Washington, D.C., 1953, chap. II, pp. 42-69.

To a technical bulletin, pamphlet, or similar publication:

<sup>4</sup> *Guidance Pamphlet in Mathematics*, The National Council of Teachers of

Mathematics, Washington, D.C., 1953, p. 16.

4. If a bibliography is included at the end of the article, use the following guide:

For a book:

CLARK, CHARLES E. *An Introduction to Statistics*. New York: John Wiley & Sons, Inc., 1953.

For an article in a periodical:

WILSON, JACK D. "Arithmetic for Majors?" THE MATHEMATICS TEACHER, Vol. XLVI (December 1953), pp. 560-64.

5. All drawings should be on good quality white paper in black india ink. Also use india ink for letters and numbers which appear in the drawing. Make drawings somewhat larger than the final print in the magazine. When mailing drawings insert cardboard in envelope to prevent bending.

6. The order for reprints must be placed when the galley proofs are returned to the editorial office. A blank for ordering reprints is furnished with each set of galley proofs. Authors of feature articles will receive four copies of the magazine from the editorial office.

7. Changes made in galley proofs that are sent to the author for proofreading are expensive. Indicate only corrections to make the galley proofs agree with your manuscript.

8. Return galley proofs promptly.

### Are you writing a manuscript?

In the September issue of the *Newsletter* of The Educational Press Association of America, we find the following suggestions for those who aspire to write. These suggestions were taken from a Monthly Letter of the Royal Bank of Canada, Montreal.

1. Before you write for publication, consider three points: Your purpose in writing; the reader's purpose in reading; and the publisher's purpose in publishing. If what you have in mind satisfies those three purposes—then write!

2. Nothing is particularly difficult if you break it down into small jobs. Writing lends it-

self admirably to such treatment.

3. A simple slogan to remember is: "Have something to write; write it; end it." Another: "Use enough details so your reader will know what the article is about." (You can carry brevity too far.)

4. To make it easy for your reader—stick to your subject.

5. Where does vividness come from? No essayist is worth his salt who does not keep fresh in his mind that the revealing incident and the illuminating flash are far, far better than the roll of drums in conveying ideas.

## ● HISTORICALLY SPEAKING,—

*Edited by Phillip S. Jones, University of Michigan, Ann Arbor, Michigan*

### **Tangible arithmetic II: the sector compasses<sup>1</sup>**

*By Florence Wood, Logansport, Indiana*

#### INTRODUCTION

The instrument pictured in Figures 1 and 2 was composed of two rulers, usually brass, hinged, with three or more scales radiating from the pivot on each side of each ruler. It takes its name from the sector in geometry which Euclid defines in Book Three as "A sector of a circle is the figure contained by two straight lines drawn from the centre, and the circumference between them."<sup>2</sup> Authorities disagree on the origin of the compasses which are sometimes called the sectorial scale of equal parts or the compasses of proportion. It is usually credited to Galileo who claimed that he invented it about 1604. Actually the compasses date back to 1568 when the instrument was invented by Guido Ubaldo del Monte. It has also been stated that Gaspar Mordente of Antwerp described it in 1584, and attributed its invention to his brother, Fabricius, in 1554. This device was also described by several German and English writers in the sixteenth century.<sup>3</sup>

The compasses were used in practical mechanics for about two centuries but were not as accurate as the slide rule. They

were used in trigonometry, spherical geometry, practical astronomy, and mathematics where scales of equal parts, chords, sines, etc., were needed. The principal use of the compasses is the graphical determination of unknown quantities by means of proportions.

TRACTATUS  
De  
**PROPORTIONVM**  
INSTRUMENTO,  
Quod merito Compendium vniuersæ  
Geometriæ dixeris,

*Autore*  
**GALILÆO GALILÆI,**  
NOBILI FLORENTINO,  
PHILOSOPHO ET MATHEMATICO  
EXCELLENTISSIMO;

*Ex Italica Lingua Latine conuersus, adiectis  
Notis, quibus & artificiosa Instrumenti fabrica,  
& vsus vltior exponitur.*



**ARGENTORATI,**  
Typis DAVIDIS HAUTTI,  
M. DC. XXXV.

Figure 1

<sup>1</sup> "Tangible Arithmetic I: Napier's and Genaille's Rods" appeared in *THE MATHEMATICS TEACHER* (November 1954), pp. 482-487. For notes on "Tallies," "Counters," "The Chinese Abacus," see *THE MATHEMATICS TEACHER* (October, November, December, 1950), pp. 292, 368, and 402 respectively.

<sup>2</sup> Isaac Todhunter, *The Elements of Euclid* (London: 1948), pp. 175-176.

<sup>3</sup> *Encyclopedia Americana* (New York: Americana Corp., 1951).

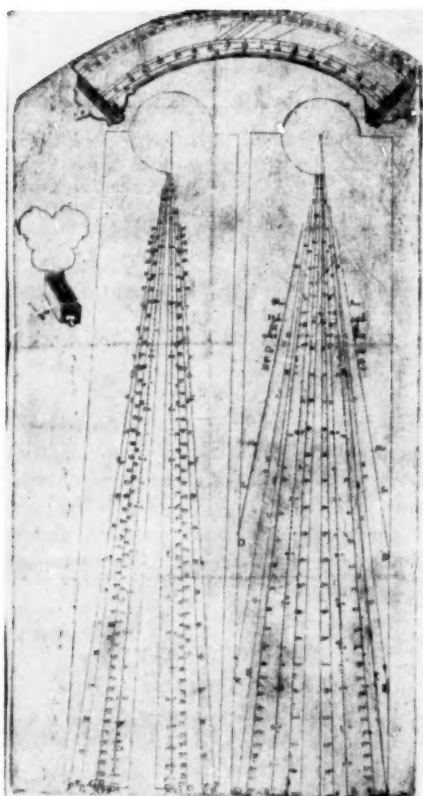


Figure 2

The sector compasses are founded on the second and fourth propositions of Book Six of Euclid where it is proved that equiangular triangles have homologous sides proportional.

Let  $AB$  and  $AC$  represent the legs of the sector (Fig. 3).  $AB = AC$ . If  $AE = AD$ , then  $AC$  and  $AB$  are cut proportionally. Draw  $BC$  and  $DE$ .  $AB:AD = BC:DE$ . If  $AB$  and  $AC$  are doubled, tripled, etc.,  $BC$  and  $DE$  are doubled, tripled, etc. Complete proofs of any theorem mentioned here may be found in Euclid's *Elements*.

The scales of Gunter's version are divided into seven general and five particular scales.<sup>4</sup> The general scales are the lines of lines, superficies, solids, sines and

chords, tangents, secants, and meridians. The particular scales are lines of quadrature, segments, inscribed bodies in the same sphere, equated bodies, and metals. Sometimes the scales are divided into single and double scales and other scales are included besides the ones mentioned above. The line of lines is the basis for the rest of the lines.

Authorities disagree on the importance of the various scales since both the scales on the compasses and their importance varied according to the use to be made of the instrument. This article is concentrated on two scales, the line of lines and the line of superficies, which would be of the most interest to plane geometry students. A model of a sector compass may be made easily from cardboard, wood or other materials by high-school students for a project, teaching aid, or exhibit.

First, just a brief account of some of the other lines that might appear on the sector compasses.

The *lines of sines* are two equal lines of natural sines that appear on the same face of the two legs. The lines start from the center of the joint, as do all the scales of the compasses proper, and are equal in length to the lines of lines. They can be used to compute  $a \cdot \sin B$ .

The *lines of chords* also drawn from the center of the joint upon each leg are also equal in length to the line of lines. They were used chiefly to set the sector at any desired angular opening.

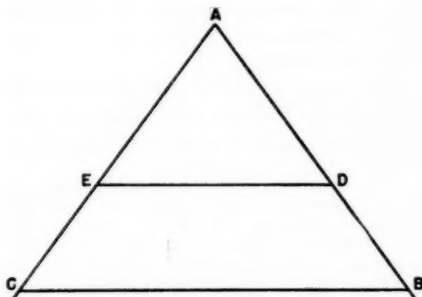


Figure 3

<sup>4</sup> Edmund Gunter, *The Description and Use of the Sector* (London: 1624), pp. 1-2.

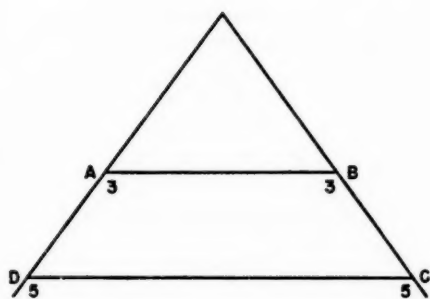


Figure 4

The *lines of tangents* are the natural tangents on lines the same length as the line of lines and running to  $45^\circ$ . The *lines of lesser tangents* are only two inches in length and run from  $45^\circ$  to  $75^\circ$ .

The *line of secants* is the same length as the line of lesser tangents but runs to about  $75^\circ$ .

Beginning at the center the *line of polygons*, numbered 4, 5, to 12, was used to construct regular polygons inscribed in circles of any given radius.

#### LINE OF LINES

The *lines of lines* or *scales of equal parts* consist of two equal lines drawn upon both legs of the sector on the same side from the center of the joint. The lines are divided into one hundred equal parts. First divide the lines into ten primary divisions and number one to ten. Then divide each of these divisions into ten equal secondary parts. The lines involved in the use of the sector were called *lateral* and *parallel*. The laterals, which are sometimes referred to as *crurals*, are measured on the sides of the sector. *AB*, *DB*, *AE*, and *EC* (Fig. 3) are called crurals. The parallels run from one leg to the other equally distant from the center of the joint. The parallels are *BC* and *DE*.

The line of lines may be used in the following ways:

1. *To increase a given line in a given proportion of 3 to 5*: Using the scale of line

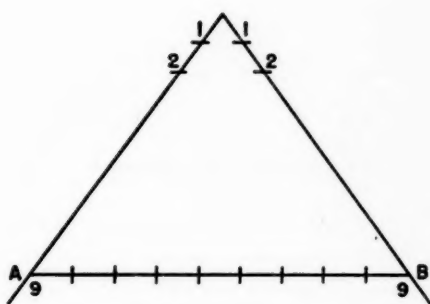


Figure 5

of lines, open the compasses so the distance *AB* is the same as the distance from the 3 to 3, on each arm (Fig. 4). Keep the compasses in the same position. *DC* is the distance from 5 to 5 on the line of lines, is the required new line, i.e.,  $AB:DC=3:5$ .

2. *To decrease a given line in a given proportion*: This problem is similar to 1.

3. *To divide a given line into any number of equal parts*: To divide line *AB* (Fig. 5) into 9 equal parts, set the opening of the compasses so that the length of the line is the distance between 9 and 9. The distance between 1-1 on the sector would be the length of one part, between 2-2 would be two parts, etc.

4. *To find the ratio between two or more lines*: To find the ratio of *AB* to *CD* (Fig. 6): Open the dividers so that the distance between 10-10 is the length of the longer line *AB*. Using the same opening find *CD* parallel to *AB*. If *CD* falls between 6-6, then  $AB:CD=10:6$ . If *AB* is too long for the opening of the sector, cut off on *AB*,  $AE=CD$ . Find the proportion of *AE* to *EB*. If this is 100 to 67, then  $AB:CD=167:100$ .

5. *To find the third proportional to two given lines*: Find the third proportional to *AB* and *AC* (Fig. 7). On arms of compasses take lengths of *AB* and *AC* and distance between arms  $BB=AC$ . Then  $AB:AC=BB:CC$ . *CC* is the third proportional.

6. *To find the fourth proportional to three given lines*: Lines *a*, *b*, and *c* are the three

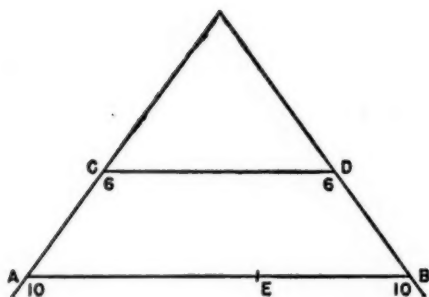


Figure 6

given lines (Fig. 8). In the compasses let  $AE = a$ ,  $AC = c$ , and  $BC = b$ . Then  $AC:AE = BC:DE$ . Line  $DE$  is the fourth proportional.

7. To divide a line in the same manner as another line is already divided: Line  $AB$  (Fig. 9) is divided at  $D$  and  $R$ . Divide  $CM$  in same manner. On the instrument  $KE = AB$ ,  $EE = CM$ ,  $KF = AD$ , and  $KH = AR$ . Then  $FF = CG$  and  $HH = CJ$ . Line  $CD$  is divided in same manner as  $AB$ .

8. To open sector so that the two lines of equal parts may make a right angle: Take three numbers that express a right triangle, say 6, 8, and 10. Measure the distance from center of sector on either line of lines to 10; then open sector so that distance between 6 on one line and 8 on other be equal to that distance. The line of lines makes right angles with each other.

The preceding eight problems are just a few ways in which the line of lines on the sector compasses may be used. These methods could easily be used in high school geometry as enrichment material for the discussion of proportional lines and similar triangles.

The following two problems are examples to illustrate practical uses of the instrument when it was widely used.<sup>5</sup>

1. Suppose the scale to the map of a survey is 6 inches long and contains 140

<sup>5</sup> John Robertson, *A Treatise of Such Mathematical Instruments as Are Usually Put into a Portable Case* (London: 1767), pp. 44-45.

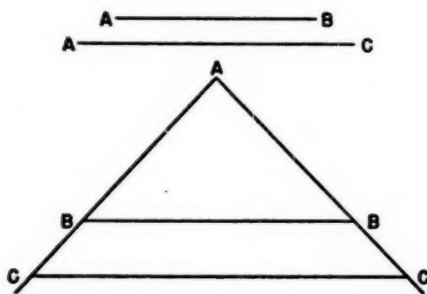


Figure 7

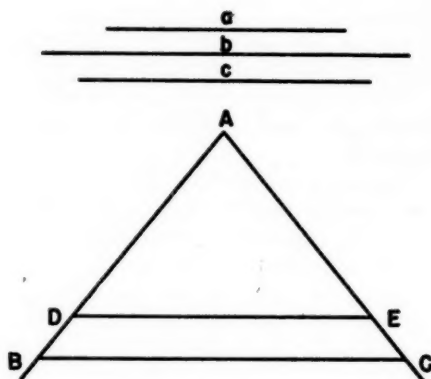


Figure 8

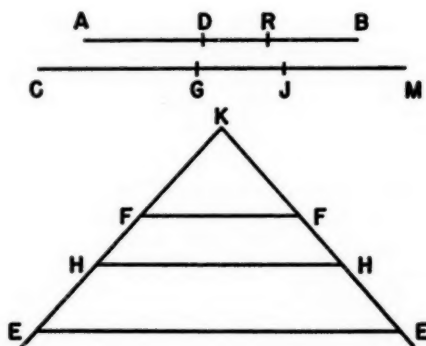


Figure 9

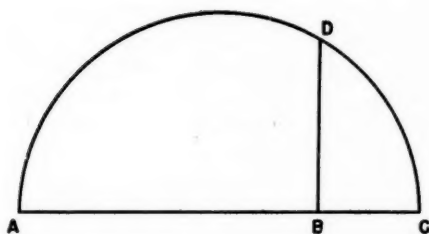


Figure 10

poles; required, to open the sector so that a corresponding scale may be taken from the line of lines. Solution: make the transverse distance 7 and 7 (or 70 and 70, viz.  $140/2$ ) equal to 3 inches ( $=6/2$ ), and this position of the line of lines will produce the given scale.

2. If it was required to make a scale of 140 poles, and to only 2 inches long. Solution: make the transverse distance of 7 and 7 equal to 1 inch, and the scale is made.

#### LINE OF SUPERFICIES

The line of superficies was used to compare areas of figures such as squares, triangles, and rectangles. This scale consisted of two equal lines drawn upon both legs of the sector on the same side from the center of the joint. The divisions of the line are based on Proposition 13 and Proposition 29, Book 6 of Euclid. Proposition 13 is: To find the mean proportional between two straight lines. Proposition 29 is: To produce a given straight line so that the rectangle contained by the segments between the extremities of the given line and the point to which it is produced may be equal to a given space.

Let  $AB$  and  $BC$  (Fig. 10) be the two given straight lines. Place them in a straight line and describe the semicircle  $ADC$ . Construct perpendicular  $BD$ .  $AB:BD=BD:BC$ . Then  $BD$  is the mean proportional between  $AB$  and  $BC$ .

In using the line of superficies the lengths of the sides of the superficies are the distance between the arms of the com-

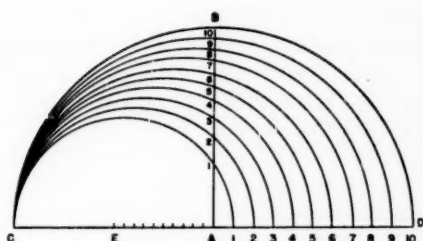


Figure 11

passes and the ratio of the areas are the numbers on the lines of superficies.

The following method is used to find the divisions (Fig. 11).

$A$  is the center of the semicircle with semidiameter equal to the line of lines. Construct  $AB$  perpendicular to diameter  $CD$ . Let  $AD$  be divided as a line of lines into one hundred parts and  $AE$ , one half of  $AC$ , also be divided into one hundred equal parts. These points will be the centers of semicircles which divide line  $AB$  into one hundred unequal parts.  $AB$  so divided is the line of superficies, and is transferred to the sector.

The line of superficies may be used in the following ways:

1. To find the proportion between two or more like superficies: Let  $a$  and  $b$  (Fig. 12) be sides of like superficies as sides of two squares  $A$  and  $B$ , respectively. Open sector so that side  $a$  is between 10-10. Keeping sector at same angle, enter side  $b$  parallel to  $a$ . If it crosses 4-4, the proportion is  $A:B=10:4$  or  $5:2$ .

2. To increase or decrease in a given proportion: Let  $a$  (Fig. 13) be side of square  $A$  to be increased in proportion of 2 to 5. Set compasses so that  $a$  is between 2-2. The parallel of 5-5 is side  $b$  on which to make square  $B$ .  $A:B=2:5$ . Decreasing would be done in a like manner.

3. To add or subtract one to another: Let  $a$  and  $b$  (Fig. 14) represent the sides of squares  $A$  and  $B$ , respectively. Find the proportion of the two squares by method in problem 1 to be  $A:B=5:2$ . Add 5 and 2

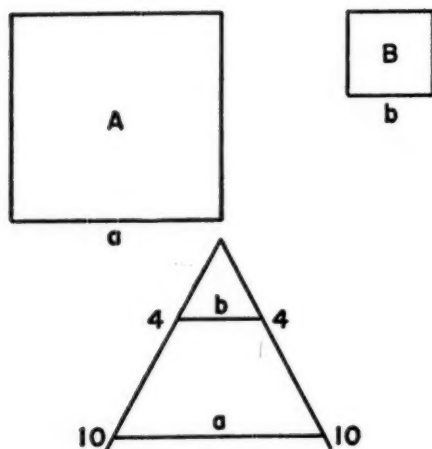


Figure 12

and increase side  $a$  in proportion of 5:7 as in problem 2, to get side  $c$ . On  $c$  construct a square which is now equal in area to the sum of the squares  $A$  and  $B$ . In like manner subtract.

4. *To find the mean proportional between two given lines:* Use the line of lines to find the proportion of line  $a$  to line  $b$  (Fig. 15). If the proportion is  $a:b=4:9$ , take line  $b$  and put it in the line of superficies 9-9. The parallel 4-4 or line  $c$  would be the mean proportion. (This, of course, amounts to finding  $c=\sqrt{ab}$ . By taking  $a=1$ , one can extract any square root.)

5. *To make a square equal to any given superficies:* If a rectangle is given find the mean proportion between the two unequal sides by method of problem 4. This is the side of the square equal to the rectangle.

6. *To find a proportion between two superficies though they be unlike one to the other:* Find the mean proportional to the unequal sides of rectangle  $A$  (Fig. 16) and call this line  $c$ . This is the side of the square equal to  $A$ . In triangle  $B$  find  $d$  the mean proportion between the altitude of  $B$  and half its base. Then  $d$  is side of square equal to  $B$ . By method of problem 1 find the proportion of the squares  $C$  and  $D$ .

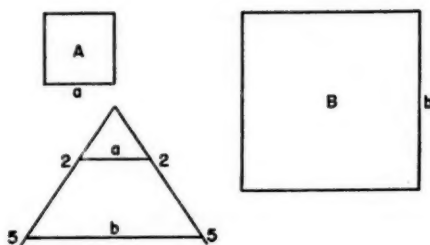


Figure 13

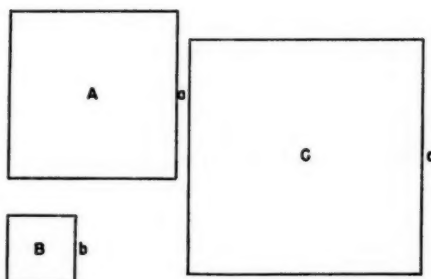


Figure 14

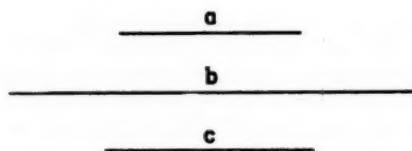


Figure 15

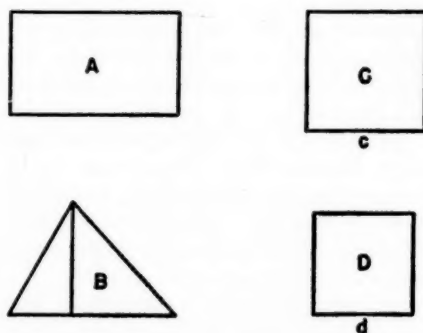


Figure 16

These are just a few of the uses of the line of superficies that would be of most interest to plane geometry students.

Examples are given rather than general explanations of the various uses of the compasses. Students could develop the proof for each problem in order to apply what they have learned concerning similar triangles and finding the mean proportional between two straight lines. These scales could be applied to interest problems, squaring numbers, extracting square root, etc., and any student particularly interested in developing a historical project could pursue the subject further.

*Editor's Note:* Miss Wood's initial summary of the history of the sector compasses indicates that there still exists a state of uncertainty and lack of clarity about its origins.

A few comments on this situation will make it possible to emphasize again several generalizations about the growth of mathematical ideas as well as to point out some unsettled questions and some unused sources which may interest a historical researcher.

The stormy and contentious life led by Galileo Galilei is well typified by the fact that he later wrote that his first publication *Le Operazioni del Compasso geometrico e militare* (Padua: 1606) would not have been published except that he heard that someone had gotten one of his instruments, pretended to be the inventor, and taught it since 1597.<sup>6</sup> More storm followed the publication because in 1607 Baldessare Capra published a description of the instrument and its use which impelled Galileo to write *Difesa . . . contro alle Calunie & Imposture di Baldessare Capra . . .* and to take his accusations of plagiarism to court where Galileo was sustained and Capra was forced to withdraw his book from sale.

<sup>6</sup> Quoted from the 1640 edition of Galileo's *Le operazioni* in J. Robertson, *op. cit.* Pages V-XVII of this book are a history of the sector compasses which does not seem to have been checked by later historians.

Zolt de Harsanyi makes a very dramatic story out of this incident in his biography of Galileo, *The Star Gazer*.<sup>7</sup>

It was Mattheus Berneggerus' translation from Italian into Latin and his additional notes which made Galileo's book widely read and understood. (A similar situation arose in connection with van Schooten's translation of Descartes' *La Géométrie* from French into Latin.) In fact Loria indicates that he was unable to find any drawings in Galileo's first edition and thinks that they may have been deliberately omitted on the assumption that a reader would have a model of the instrument in his hand and not need a drawing.<sup>8</sup> D. E. Smith, however, reproduces a drawing supposedly from Galileo's first edition with the translation of the first part of the text.<sup>9</sup> Our Figures 1 and 2 are from Berneggerus' edition.

However, the story of the sector compasses illustrates again that rarely, if ever, does a mathematical idea spring full grown from the head of an inventor. Several studies have successively shown similar instruments and ideas to have existed earlier than Galileo's claims. Favaro and Robertson relate that Marchese Guido Ubaldo del Monte saw an instrument at the home of its maker, Simone Boraccio, at Urbino, which had been designed by Commandinus for a gentleman, Bartholomew Eustachio. The Marchese suggested the use of flat legs which much improved the instrument which had been designed to divide lines into equal parts.<sup>10</sup> The original teller of this story went on to say that many copies had been made and many treatises written since 1568-1570 when this occurred.<sup>11</sup> Guido Ubaldo him-

<sup>7</sup> Zolt de Harsanyi, *The Star Gazer* (New York: G. P. Putnam's Sons, 1939), Chap. V (end), Chap. VI.

<sup>8</sup> Gino Loria, *Storia delle Matematiche*, 2nd ed. (Milan: Ulrico Hoepli, 1950), p. 409.

<sup>9</sup> D. E. Smith, *A Source Book in Mathematics* (New York: McGraw-Hill Book Co., 1929), pp. 186-191.

<sup>10</sup> A. Favaro, "Per la storia del compasso di proporzione," *Atti del R. Ist. Ven.* LXVII (1907-08), parte 2<sup>a</sup>, 723-739. J. Robertson, *op. cit.*

<sup>11</sup> Muzio Oddi, *Fabrica et uso del compasso polimetro* (Milan: 1633).

self met and aided Galileo during Ubaldo's residence in Florence as inspector of fortresses, around 1588.

Favaro conjectured, in 1907 and 1908, that the "Compasso di proportione" was derived from an earlier "compasso di riduzione" of which he could find no description. In 1931 Giuseppe Boffito published diagrams, dated 1567, of scales to be used with such an instrument and a drawing of the instrument from a 1586 work of Giordano Bruno's, both of which attributed the instrument to Fabricius Mordente.<sup>12</sup> Mordente's instrument had sliding extensible legs without scales and four pointed spurs soldered to it in pairs. It could be mounted on a tripod and used as a surveying instrument.<sup>13</sup> Although there are significant differences between Mordente's and Galileo's instruments (the latter also had a protractor type of attachment for measuring angles), they were similar in appearance and used the same principles to solve problems in proportion.

<sup>12</sup> G. Boffito, *Il primo compasso Proportionale costruito da Fabrizio Mordente . . .* (Firenze: 1931), pp. 28-33.

<sup>13</sup> Michel Connette, *L'Usage du Compas de Fabrice Mordente . . .* (Paris: 1626), and *La Geometrie reduite . . . Pantometre ou compas de Proportion de Michel Connette* (Paris: 1626). See also, Muzio Oddi, *op. cit.*

The popularity of the instrument is attested by Robertson's list of forty-four books, thirty-five written before 1700, describing it. They were written in Italian, Latin, French, English, Spanish. William Ransom notes that in this country the first edition of Bowditch's *Navigator* (1802) included a chapter on "The Sector" and that even today students occasionally bring one to their teachers.<sup>14</sup> There is one in Section M-4 of the Henry Ford Museum in Dearborn. Ransom also noted one sector which contained a logarithmic or Gunter scale parallel to an outside edge and which then could be used to extract more general roots than allowed by the usual special sectorial scales.

J. H. Lambert discussed the proportional compasses as a device for computing points in perspective drawing in his *Kurzgefaszte Regeln zu perspectivischen Zeichnungen . . .*, (Augsburg: 1768).

Some unsettled historical relations exist in the listing by Robertson of books dealing with this instrument by Speckle (1589), Hood (1598), Clavius (1604), Hulsius (1604), Horseher (1605), and Justus Burgius.<sup>15</sup>

<sup>14</sup> Wm. R. Ransom, "An Old Time Computer," *Mathematics Magazine*, 27 (1954), p. 205.

<sup>15</sup> Robertson, *op. cit.*

## Perfect numbers

On October 17, 1952, D. H. Lehmer, University of California, announced that the numbers  $2^n - 1$  for  $n=2203$  and  $2281$  were prime numbers. The SWAC computing machine enabled him to obtain these results. It took SWAC 59 minutes to show that  $2^{2203} - 1$  was a prime number.

Each Mersenne prime is associated with a perfect number, that is, a number which is equal to the sum of its divisors less than itself. Thus 6 is a perfect number because  $6=3+2+1$ . The

first four perfect numbers are 6, 28, 496, and 8128. In general  $2^{n-1}(2^n - 1)$ ,  $n=2, 3, 7, 13, 17, 19, 31, 61, 89, 107, 127, 521, 607, 1279, 2203$  and  $2281$  is a perfect number.

H. S. Uhler, Yale University, announced the discovery of the 16th and 17th perfect numbers in *Scripta Mathematica*, Vol. XIX, Nos. 2-3, 1953, p. 128. The 16th perfect number is  $2^{405} - 2^{2202} = 2^{2202}(2^{2203} - 1)$  and the 17th is  $2^{281} - 1$ . Each of these numbers consists of 1327 and 1372 digits respectively (base 10).

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Mathematics involves the development of the most precise, most sustained, most closely knit set of habits.—Henry Link, *The Return to Religion*, Macmillan, 1937, p. 153

## ● DEVICES FOR THE MATHEMATICS CLASSROOM

*Edited by Emil J. Berger, Monroe High School, St. Paul, Minnesota*

### *Models of loci*

*By John F. Schacht, Bezley High School, Bezley, Ohio*

**EDITOR'S NOTE:** While attending the annual meeting of The National Council of Teachers of Mathematics at Cincinnati last April, this department editor had the opportunity to see some rather unusual models and devices. Among the most interesting of those displayed were the loci models presented in this issue. These were designed by John Schacht of Bezley, Ohio. As some readers may know, John is one of the pioneers whose early work with models and devices is a contributing reason why this department came into being.—E. J. B.

Determining the location of points that satisfy certain prescribed conditions has always been an interesting topic in the study of mathematics. Constructing models of loci should help students develop an appreciation of the concept that a "locus of points" is in general some geometric figure, a fact which oftentimes remains quite obscure for beginning students. The models presented here are useful devices for helping students clarify their understandings about some of the ideas usually considered in beginning work with loci.

Of the models described, some are fixed figures resulting from loci constructions; others are dynamic devices which indicate how certain figures may be generated. Contrary to what one might expect, the fixed figures, and those that are partially fixed, have uses that make a greater appeal to the imaginations of students than those that may be manipulated more fully and which illustrate the manner in which certain figures are generated. However, before we plunge ahead with this idea let us examine the construction of the models to which we have reference. As an expedient for avoiding confusion, the particular

locus of points which is illustrated or which can be generated by an individual model is printed below the model to which it refers.

The models shown in Figures 1 to 3 may be constructed from strips of colored plastic, thin sheets of transparent plastic, or pieces of colored cardboard. The dark shaded parts indicate the *given* parts of a figure. In the finished models these parts should be blue in color. The lightly shaded parts indicate the *locus of points* in the different figures; these parts should be red in the finished models. The open or unshaded parts are transparent strips (assuming that the material used is plastic) or strips of white cardboard which hold the rings in place. If the models are constructed with plastic, it seems appropriate to mention that strips  $\frac{1}{4}$ " wide cut from sheets  $\frac{1}{4}$ " thick will produce nice results. For one not familiar with plastic it should also be mentioned that the strips used for the circles must be heated, shaped to form a loop, and joined at the ends with cement. (Model airplane glue will work satisfactorily with most types of plastic material.) The circles may be held in place by fastening them to the transparent strips with drive screws. If transparent sheets of plastic are used, it is only necessary to stencil the *given* (blue) and *locus of points* (red) for any one figure on the same sheet. When this construction is employed, no transparent strips are needed. The mode of construction with cardboard should be obvious. The diameter of the circle for the

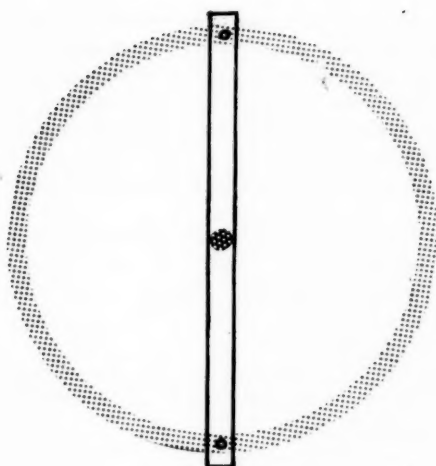


Figure 1

*The locus of points at a given distance from a given point.*

model in Figure 1 should be 6", and for the models in Figures 2 and 3, the diameters should be, respectively 4", 6", and 8".

The same convention for representing *given parts* and the *locus of points* is recommended for the models in Figures 4 and 5 as was indicated for the first three models described. The construction of the models in Figures 4 and 5 should be planned so that the completed device may be extended or collapsed as desired. The horizontal strips should be 18" long and the cross strips should each be 9" long, with the outside horizontal strips fastened  $\frac{1}{2}$ " from the extremities of the cross strips. The models in Figures 4 and 5, as well as the one in Figure 6, may be made with plastic strips having cross-section dimensions of  $\frac{1}{4} \times \frac{1}{4}$ ". Colored cardboard strips may also be used. The strips used for the model in Figure 6 should each be about 13" long.

Figure 7 is a dynamic model in which the two *given points* are painted on either a strip of clear plastic or a white wooden dowel. The *locus of points* is represented with a red dowel. The ring clamp is a loop that fits over the red dowel; its position

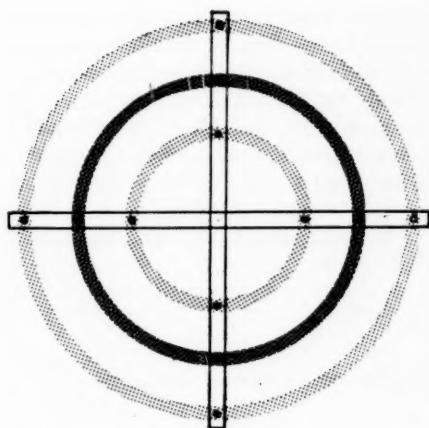


Figure 2

*The locus of points at a given distance  $d$  from a given circle of radius  $r$  when  $d$  is less than  $r$ .*

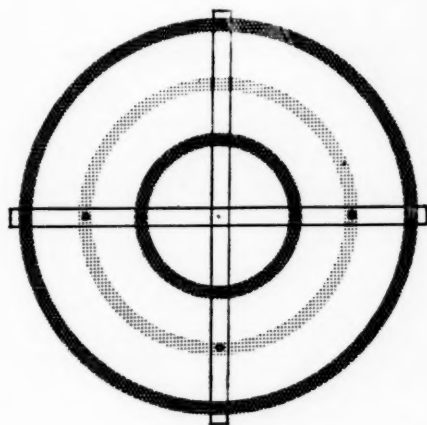


Figure 3

*The locus of points equidistant from two given concentric circles.*

may be fixed at any point along the length of the dowel by tightening the wing nut. The black lines are black elastic.

The models presented thus far are useful for illustrating concretely what some of the fundamental loci are, but, as was indicated earlier, they have another use—one that has proved itself to be exceedingly effective. By way of illustration we state an

example: What is the locus of points at a given distance from a given point, and at a second given distance from a given circle ( $d < r$ )? A moment's reflection should bring one to the realization that this problem has more than one solution. There are, in fact, six possibilities. The reader should

satisfy himself that this is true. (Completely disjoint figures and concentric circles are both considered as separate possibilities even though there are no points satisfying the compound locus in either case.)

The models illustrated in Figures 1 and



Figure 4

*The locus of points equidistant from two given parallel lines.*

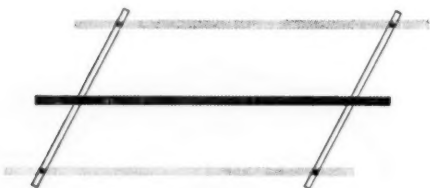


Figure 5

*The locus of points at a given distance from a given line.*

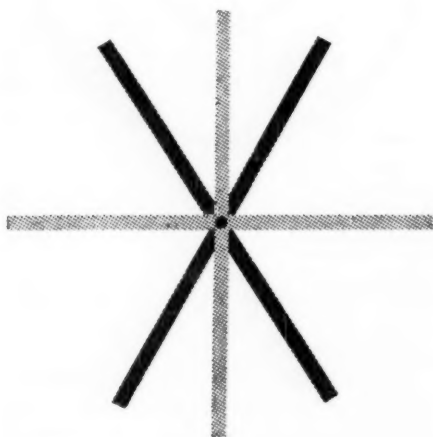


Figure 6

*The locus of points equidistant from two intersecting straight lines.*

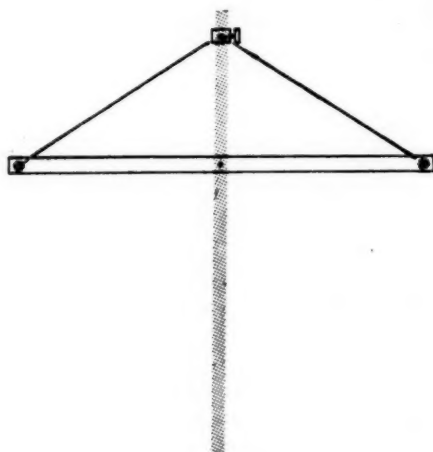


Figure 7

*The locus of points equidistant from two given points.*

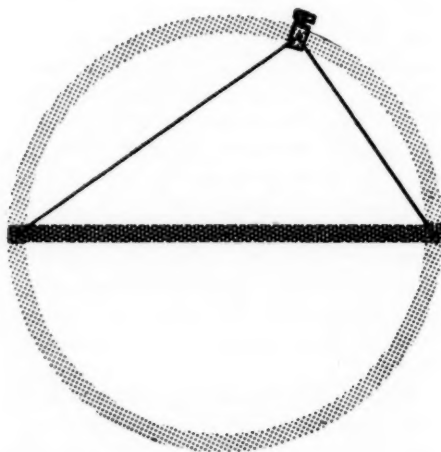


Figure 8

*The locus of the vertex of the right angle of all right triangles having a given hypotenuse.*

2 provide an effective means for developing the intricacies of the problem posed above. It hardly seems necessary to mention that by means of the models illustrated in Figures 1 to 7, a whole host of exercises similar to the one already proposed may be planned for consideration by one's students.

The diagrams shown in Figures 8 to 11 indicate how dynamic models may be used to give concrete meaning to the concept that a moving point generates a geometric figure. In this respect they are similar to the model illustrated in Figure 7. The *given* parts and the *loci* are again represented by differences in shading. Since sturdiness is a requirement for these particular models the reader may find it to his advantage to construct them by using ordinary steel rims (about 8" in diameter) for the circles and steel strap iron or wooden dowels for the straight bars. The narrow lines may be produced with black elastic.

Probably every teacher who has read the preceding description has at some time

felt the need of a scheme for putting across the concept of locus. Establishing communication with one's students seems always to be a problem with this topic. While the writer has found the method presented in this article satisfyingly effective, he would not go so far as to recommend it as a standard procedure. The writer hopes only that the publication of his ideas will serve to provide hints of possibilities that the reader may find to his liking.

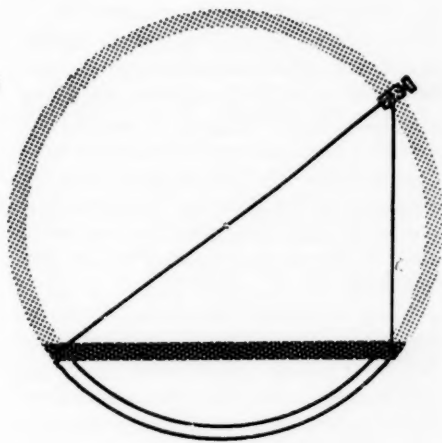


Figure 10

*The locus of the vertex of a triangle having a given base and a given vertex angle.*

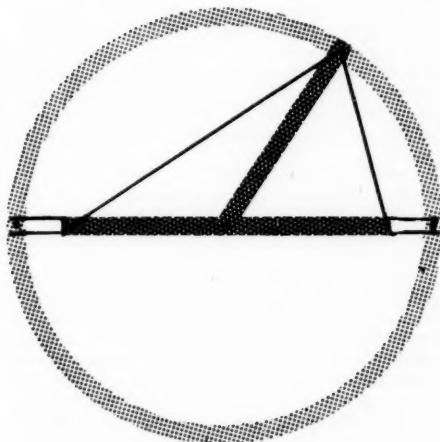


Figure 9

*The locus of the vertex of a triangle having a given base and a given median to the base.*

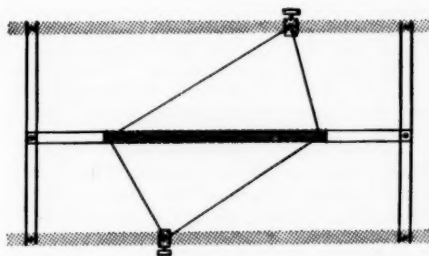


Figure 11

*The locus of the vertex of a triangle having a given base and a given altitude to that base.*

## ● POINTS AND VIEWPOINTS

*A column for unofficial comment*

### *Who does the work?—NCTM committee structure*

*By Marie S. Wilcox, President, The National Council of Teachers of Mathematics*

A large organization succeeds only to the extent that its members are willing to donate their time and talents to participate in the activities of that organization. Important among the activities of the National Council are those of its several committees. The following committees have been appointed by the president with the approval of the Board of Directors. Appointments are for a period ending at the close of the annual meeting of the year indicated.

Many other members of the Council are serving on committees associated with the Affiliated Groups, and on local committees which assist with the arrangements for meetings of the Council. These committees are not listed here.

#### *National Council business*

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**AUDITING** (1955): Russell B. Coover, Chevy Chase, Md.; Jane M. Hill, Washington, D. C.

**BUDGET**: Donovan Johnson, Minneapolis, Minn., Chairman (1955); John R. Mayor, Madison, Wis. (1957); Irene Sauble, Detroit, Mich. (1956)

**CONVENTIONS** (1955): Irene Sauble, Detroit, Mich., Chairman; M. H. Ahrendt, Washington, D. C.; Harry Charlesworth, Denver, Colo.; W. A. Gager, Gainesville, Fla.; Agnes Herbert, Baltimore, Md.; Houston Karnes, Baton Rouge, La.; Mildred Keiffer, Cincinnati, O.; Mary C. Rogers, Westfield, N. J.

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Junge, Detroit, Mich.; M. Albert Linton, Jr., Philadelphia, Pa.; H. Vernon Price, Iowa City, Ia.

**EXECUTIVE**: Marie Wilcox, Indianapolis, Ind., Chairman (1956); H. Glenn Ayre, Macomb, Ill. (1955); Harold Fawcett, Columbus, O. (1955)

**COOPERATION WITH FSAF** (1956): Donald W. Lentz, Parma, O., Chairman; Glenadine Gibb, Cedar Falls, Ia.; George E. Hawkins, LaGrange, Ill.; Phillip S. Jones, Ann Arbor, Mich.; Veryl Schult, Washington, D. C.

**MEMBERSHIP DRIVE** (1956): Mary Rogers, Westfield, N. J., Chairman; M. H. Ahrendt, Washington, D. C.; Pearl Bond, Beaumont, Tex.; Lucy Hall, Cheney, Kansas; Janet Height, Wakefield, Mass.; Harold Hunt, Seattle, Wash.; Nelle Kitchens, Columbia, Mo.; Ona Kraft, Cleveland, O.; L. Clark Lay, Pasadena, Calif.; Bess Patton, Atlanta, Ga.; Mary Reed, Benton Harbor, Mich.; Alice M. Reeve, Rockville Center, N. Y.

**RELATIONS WITH NEA**: Kenneth E. Brown, Washington, D. C., Chairman (1955); Allene Archer, Richmond, Va. (1955); Ida May Bernhard, Austin, Tex. (1956); Harry Charlesworth, Denver, Colo. (1955); W. A. Gager, Gainesville, Fla. (1956); Mabel Milldollar, Tarentum, Pa. (1956); Ethel McDonough, Cincinnati, O. (1956)

**NOMINATION PROCEDURE** (1955): Allene Archer, Richmond, Va., Chairman; M. H. Ahrendt, Washington, D. C.; Agnes Herbert, Baltimore, Md.; Howard F. Fehr, New York, N. Y.

**NOMINATIONS FOR EDITOR OF STUDENT JOURNAL** (1955): Phillip S. Jones, Ann Arbor, Mich., Chairman; Clela Hammond, El Camino, Calif.; Lesta Hoel, Portland, Ore.; Margaret Joseph, Milwaukee, Wis.; Henry Van Engen, Cedar Falls, Iowa

**PLACE OF MEETING**: Elizabeth Roudeshush, Seattle, Wash., Chairman (1956); Alice Hach, Ann Arbor, Mich. (1957); Agnes Herbert, Baltimore, Md. (1955); Houston Karnes, Baton Rouge, La. (1956); Joseph N. Payne, Helena, Ala. (1957); Edith Woolsey, Minneapolis, Minn. (1955)

**PUBLICITY (1956):** James Zant, Stillwater, Okla., Chairman; M. H. Ahrendt, Washington, D. C.; Helen Alphin, Denver, Colo.; Edwin Eagle, San Diego, Calif.; Gilbert Ulmer, Lawrence, Kans.

#### **PUBLICITY SUBCOMMITTEES**

**POPULAR MAGAZINES:** John R. Clark, New York, N. Y., Chairman; Edwina Deans, Arlington, Va.; Howard F. Fehr, New York, N. Y.; John R. Mayor, Madison, Wis.

**JOURNALS OF EDUCATION:** Milton W. Beckman, Lincoln, Neb., Chairman; Mabel Baker, Verona, Pa.; Harriet Burr, San Jose, Calif.; Eunice Lewis, Norman, Okla.; Ella Marth, Washington, D. C.

**RECOGNITION OF RETIRED MEMBERS (1955):** Charles Butler, Kalamazoo, Mich., Chairman; Mamie Auerbach, Richmond, Va.; Walter H. Carnahan, Lafayette, Ind.; H. C. Christoffer-son, Oxford, O.; R. R. Smith, Springfield, Mass.

**SUPPLEMENTARY PUBLICATIONS:** Henry W. Syer, Boston, Mass., Chairman (1955); J. Houston Banks, Nashville, Tenn. (1957); Lawrence P. Bartnick, Natick, Mass. (1957); Barbara Bets, Swanscott, Mass. (1956); Hope Chipman, Ann Arbor, Mich. (1955); Donovan Johnson, Minneapolis, Minn. (1955); Phillip S. Jones, Ann Arbor, Mich. (1955); Margaret Joseph, Milwaukee, Wis. (1956); Rachael Keniston, Stockton, Calif. (1957); Ann Peters, Keene, N. H. (1955); Vera Sanford, Oneonta, N. Y. (1957); Henry Swain, Winnetka, Ill. (1955)

**YEARBOOK PLANNING:** Daniel Snader, Urbana, Ill. (1955), Chairman; Harold Fawcett, Columbus, Ohio (1957); F. G. Lankford, Charlottesville, Va. (1956)

#### **Other professional activities**

**ADVISORY ON DISTRIBUTION OF FREE MATERIALS (1955):** John A. Brown, Madison, Wis., Chairman; Edwina Deans, Arlington, Va.; Howard F. Fehr, New York, N. Y.; Roderick McLennan, Winnetka, Ill.; Sheldon Myers, Columbus, O.

**CONTESTS, SCHOLARSHIPS, TALENT SEARCH (1956):** Daniel B. Lloyd, Washington, D. C.,

Chairman; Robert S. Fouch, Tempe, Ariz.; Franklin Frey, Detroit, Mich.; Jeannette Garrett, Birmingham, Ala.; Mary Lee Foster, Arkadelphia, Ark.; Frances Johnson, Oneonta, N. Y.; Sylvia Vopni, Seattle, Wash.

**COOPERATION OF MATHEMATICS WITH INDUSTRY (1956):** W. W. Rankin, Durham, N. C., Chairman; Clifford Bell, Los Angeles, Calif.; Nanette R. Blackiston, Baltimore, Md.; Kenneth Brown, Washington, D. C.; Paul C. Clifford, Montclair, N. J.; Phillip S. Jones, Ann Arbor, Mich.; Catherine Lyons, Pittsburgh, Pa.; C. V. Newsom, Albany, N. Y.; Myron Rosskopf, New York, N. Y.; R. M. Thrall, Ann Arbor, Mich.; W. M. Whyburn, Chapel Hill, N. C.

**NATIONAL HONORARY SOCIETY FOR STUDENTS (1955):** Richard V. Andree, Norman, Okla., Chairman; Eileen Beckett, Lebanon, Ind.; Ida Mae Heard, Lafayette, La.; Virginia Lee Pratt, Omaha, Neb.; Eugene P. Smith, Columbus, O.

**INTERNATIONAL RELATIONS (1956):** Howard F. Fehr, New York, N. Y., Chairman; Lilla C. Lyle, Miami, Fla.; Veryl Schult, Washington, D. C.; Henry W. Syer, Boston, Mass.; Lauren Woodby, Mt. Pleasant, Mich.

**RESEARCH COMMITTEE:** Bruce E. Meserve, Montclair, N. J., Chairman (1955); Kenneth Brown, Washington, D. C. (1957); Maurice L. Hartung, Chicago, Ill. (1957); E. H. C. Hildebrandt, Evanston, Ill. (1956); Esther Swenson, University, Ala. (1956); Harold Trimble, Cedar Falls, Ia. (1955)

**TEACHER EDUCATION IN MATHEMATICS—**Jointly with MAA (Jan. 1, 1955): Carroll Newsom, Albany, N. Y., Chairman; Burton Jones, Boulder, Colo.; Kenneth May, Northfield, Minn.; Robert Pingry, Urbana, Ill.; Alfred Putnam, Chicago, Ill.; Henry Van Engen, Cedar Falls, Iowa; Robert Yates, West Point, N. Y.

**TELEVISION (1955):** L. F. Scholl, Buffalo, N. Y., Chairman; Kenneth Brown, Washington, D. C.; Esther Gibney, Houston, Tex.; Herschel Grime, Cleveland, O.; Arthur J. Hall, Menlo Park, Calif.; Mildred Keiffer, Cincinnati, O.; John K. Reckzeh, Jersey City, N. J.; Veryl Schult, Washington, D. C.; Sylvia Vopni, Seattle, Wash.

## **Have you read?**

GATTEGNO, D. "The Use of Mistakes in the Teaching of Mathematics." *The Mathematical Gazette*, Greenwich, England, February 1954, pp. 11-14.

How effectively are you making use of the mistakes in mathematics that are made by your students? Do these mistakes point out significant characteristics of our teaching? Is the fact that certain types of errors continue to recur due to dullness on the part of the learner? These ques-

tions are carefully considered and some very pertinent points are clarified in terms of the teacher's responsibility and his efficiency. The author also presents some valuable aids in creating an awareness in the pupil of mental processes. This article will make you wonder whether or not you have been properly directing your efforts.—*Philip Peak, Indiana University, Bloomington, Indiana.*

# Notes from the Washington office

by M. H. Ahrendt, Executive Secretary

## The National Council is big business

It is likely that very few members of The National Council of Teachers of Mathematics have a real conception of how big and complex their organization is. If you compare the Council with many other similar organizations, you will realize that the Council is rapidly becoming "big business." You may be interested in seeing part of our financial picture and learning how our funds are handled.

The over-all financial picture of the Council is impressive. Our total receipts during the fiscal year ending May 31, 1954, were \$64,000, an increase of 60% within two years. Our total expenditures were just under \$64,000. Thus during the past fiscal year, the Council suffered no losses but made no significant financial gains. In the face of rising prices, it may be harder to "break even" in the future.

Anyone who explores the complexities of our financial operations will soon realize that our bank balance is a very incomplete indicator of our financial condition. First, our bank balance represents only part of our assets. Second, not all our cash funds are expendable. For example, on May 31, 1954, we had in the bank about \$13,700 that we had received in payment for uncompleted subscriptions to our three journals. This money actually belonged to our subscribers and had to be held to complete the service for which they had paid. For another example, our publication sales during the past fiscal year totalled nearly \$16,000. But the increase in our expendable assets was much less than this, for some of this money had to be held to replace the books and pamphlets that had been sold. Otherwise we should soon sell ourselves out of business.

In order to get a more complete analysis of our total financial condition, the executive secretary set up a system more than a year ago by which we can calculate periodically our net worth. We now prepare a Statement of Assets and Liabilities. This includes, along with our cash assets, all money due us for services rendered, and the cost value of our inventory of publications (less an appropriate reserve for obsolescence). Among our liabilities we include the bills we owe others and the amount that must be reserved to meet our obligations to our subscribers. Our net worth is then the difference between our total assets and our total liabilities. The figure for any given date is not highly meaningful. But changes in the net worth are of vital importance. A gain in net worth means a net profit. A decline means a net loss. A continuing decline would mean that we are on the road to bankruptcy. During the past fiscal year, our net worth remained practically constant.

The funds of the Council are divided into two main accounts, the operating account and the publications account. The operating account provides for the general operation of the Council. All receipts from the sale of memberships, subscriptions and advertising in the journals, are deposited in this account. All expenditures for office costs, printing of the journals, committee work, and other operating costs are charged to it. All receipts from the sale of publications are deposited in the publications account, and all expenses for the printing of publications are charged to this account.

The operating account sustains a loss each year, since the operating receipts are

not sufficient to take care of the operating expenses. One solution, of course, would be to increase the operating income by raising the membership and subscription fees. Instead, the Board of Directors has preferred to push the sale of publications, in the hope that the publications account will make enough profit to balance each yearly budget. Thus, when you purchase the yearbooks and other publications of the Council,

you help the Council as well as yourself.

This brief description of some of our accounting procedures should make it clear that every attempt is being made to operate the Council on a sound financial basis. We know precisely where our money is coming from and where it is going. We are glad to know that at the present time the Council is in sound financial condition.

### **How your subscription is processed**

A matter of importance to every subscriber to *THE MATHEMATICS TEACHER* is the process by which memberships and subscriptions are handled. If something goes wrong with the process, the issues of *THE MATHEMATICS TEACHER* are not delivered on schedule, and we have a disappointed customer.

No doubt the most important characteristics of a good subscription process are accuracy, efficiency, and speed. The members of the staff in the Washington office have given much attention and effort to the problem of developing good procedures. Let us trace the subscription process from the time your order is mailed to the time that regular service begins on your subscription to *THE MATHEMATICS TEACHER*.

1. Your letter must first be delivered to us by the post office.

2. After the mail has been sorted, the subscription orders are checked against the files. The subscription files consist of metal plates upon which your name and address are embossed. The plate bears a card insert upon which we post the expiration date and the payment record. The plates are filed under the address, geographically by states and cities, as required by postal mailing regulations. If the order is for the renewal of a current subscription, the process is quick and simple. We merely record the new date and drop the plate back into its place in the file.

3. If the order is for the renewal of a recently expired subscription, the plate will have been removed to the "drop" files. In this case the subscriber will be entitled to one or more back issues, and the plate will be removed temporarily to a "back-issue" file.

4. If the order is new, the order is typed with others on sheets of paper and sent to the Records Division of the NEA to have a new plate embossed. These new plates are filed in the current file or in a "back-issue" file depending on whether mailing is to begin with the current issue or a back issue. From this point on, the process is largely automatic.

5. The issues of *THE MATHEMATICS TEACHER* are mailed by the printer in Menasha, Wisconsin. The plates are printed by an automatic machine on a long strip or "tape" of paper. The strip for one mailing, bearing nearly 10,000 addresses, makes a roll about 8" in diameter. The printer uses a hand machine to paste the addresses on the wrappers for *THE MATHEMATICS TEACHER*.

6. Plates that are to receive back issues are printed on a tape by a hand machine in the Council office. After the printer has mailed the back issues requested, these plates are sorted into the current file for the next main mailing.

7. If the customer gave us the correct address, no slip-ups occurred, and the mail service was good, the issue was delivered on schedule. The complete subscription

process takes about four weeks. We have been trying to find ways to shorten it.

There are many places in the process where an error *could* occur. We, you customers, the printer, and the post office all make mistakes. Everything considered, the surprising fact is that the total number of errors is so small.

About six weeks before a subscription is to expire, a renewal notice is sent. Later, if no renewal payment has been received, a second notice is sent. The renewal notices are addressed by machine directly from the plates and are sent third-class in window envelopes, thus saving much in labor and postage.

The subscription process that we use is

the fastest and most fool-proof one that we have been able to devise to date. Experience, and your suggestions perhaps, may enable us to improve it further. The metal plate system for the files is especially flexible. It makes it possible for us, with very little labor, to run off special membership lists or to make special mailings to our membership.

One fact in particular should be clear from the above material. When writing to us for any reason about your subscription, be sure to give us the mailing address that we have in the files. It may be impossible for us to find your plate without it. And be sure to return your renewal notice when you renew.

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## Your professional dates

The information below gives the date, name, and place of meeting, with the name and address of the person to whom you may write for further information. Except for NCTM meetings, each date can be listed only twice. For information about other meetings, see the previous issues of

### NCTM convention dates

December 27-29, 1954

#### CHRISTMAS MEETING

Chase Hotel, St. Louis, Missouri  
Jesse Osborn and Margaret Willerding, local chairmen, Harris Teachers College, 5351 Enright Avenue, St. Louis 12, Missouri

April 13-16, 1955

#### ANNUAL MEETING

Statler Hotel, Boston, Massachusetts  
Jackson Adkins, local chairman, Phillips Exeter Academy, Exeter, New Hampshire

July 4, 1955

#### JOINT MEETING WITH NEA

Chicago, Illinois  
E. H. C. Hildebrandt, local chairman, Northwestern University, Evanston, Illinois

August 21-24, 1955

#### SUMMER MEETING

Indiana University, Bloomington, Indiana  
Philip Peak, local chairman, Indiana University, Bloomington, Indiana

**THE MATHEMATICS TEACHER.** Announcements for this column should be sent at least two months early to the Executive Secretary, The National Council of Teachers of Mathematics, 1201 Sixteenth Street, N. W., Washington 6, D. C.

### Other professional dates

April 29-30, 1955

Fifth Annual Spring Conference of the Ohio Council of Teachers of Mathematics  
Kent State University, Kent, Ohio  
Miss Mildred Keiffer, Cincinnati Public Schools, Cincinnati, Ohio

May 6-7, 1955

Annual Meeting of Association of Mathematics Teachers of New York State  
Hotel Syracuse, Syracuse, New York  
Pauline A. Morris, Geneva High School, Geneva, New York

July 6-15, 1955

Third New Jersey Mathematics Institute  
Rutgers University, New Brunswick, New Jersey  
Director of the Summer Session, Rutgers University, New Brunswick, New Jersey

August 18-25, 1955

7th Annual Institute for Teachers of Mathematics (sponsored by Association of Teachers of Mathematics in New England)  
Middlebury College, Middlebury, Vermont  
Harriet Howard, Ethel Walker School, Simsbury, Connecticut

## ● AFFILIATED GROUPS

*Edited by H. Glenn Ayre, Western Illinois State College, Macomb, Illinois*

### *A message to Affiliated Groups*

*By H. Glenn Ayre, Chairman, Affiliated Groups*

The key role played by Affiliated Groups in the work of the National Council was demonstrated at the Fourteenth Summer Meeting of the National Council held at Seattle, Washington, August 22-25, 1954. Over 100 members of the recently affiliated Puget Sound Council of Teachers of Mathematics served on committees for local arrangements and provided superb hospitality of Western quality for visiting teachers. Sections of the program were sponsored by the Southern Oregon Council of Teachers of Mathematics, the Illinois Council of Teachers of Mathematics, and the Puget Sound Council of Teachers of Mathematics.

A project of immediate interest to all Affiliated Groups is the planning for the Sixth Delegate Assembly at the annual meeting of the National Council in Boston, April 13-16, 1955. Dr. Catherine A. V. Lyons, 12 So. Fremont Ave., Pittsburgh 2, Pennsylvania, has agreed to serve as chairman for the Agenda Planning Committee; she will need your help. Please write any suggestions as to problems or topics that should be discussed by this Delegate Assembly, to the chairman of the Agenda Committee or to your regional representative. The regional representatives are:

Northeastern area: Jackson B. Adkins,  
Phillips Exeter Academy, Exeter, New  
Hampshire

Southeastern area: William A. Gager,  
University of Florida, Gainesville, Florida

North Central area: Irene Sauble, 467 W.  
Hancock, Detroit 1, Michigan

Southwestern area: Ida May Bernhard,  
Texas Educational Agency, Austin,  
Texas

Northwestern area: Elizabeth Roudebush,  
815 4th Avenue North, Seattle 9, Wash-  
ington

The Renewal of Affiliation forms will soon be sent to each Affiliated Group. At the annual meeting of the Council in Cincinnati the Board of Directors voted to have all financial accounts of the Council centralized in the office of the Executive Secretary. Consequently, the affiliation dues will now be sent direct to the Executive Secretary.

Since the *Newsletter for Affiliated Groups* will not appear as a separate publication this year, all groups are urged to make a special effort to mail copies of all publications to all officers of Affiliated Groups including the regional representatives and the general chairman. This method of intercommunication will let all groups know about your programs and projects and be of general mutual benefit. Any special items of general interest should be sent to your regional representative for possible publication in *THE MATHEMATICS TEACHER*.

The committee working on the *Handbook for Affiliated Groups* has revised the material as presented to the Fifth Delegate Assembly in Cincinnati. Mr. Henry Swain, New Trier Township High School, Winnetka, Illinois, has been appointed editor

to replace H. Glenn Ayre. The Executive Committee of the Board of Directors has recommended that a preliminary edition of the proposed handbook be presented in mimeographed form so it can be used on a trial basis before being printed in final

form. The committee will probably complete the work on the manuscript at the St. Louis meeting of the Council and have the preliminary edition ready to present to the Sixth Delegate Assembly when they meet in Boston.

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### Have you read?

SALVADORI, MARIO. "Math's a Pleasure." *Harper's Magazine*, August 1954, pp. 88-91.

You will be interested in this engineer's reminiscing on his mathematics experiences, as well as his interpretation of how such experiences affect a student. For example: what does it do to you when a problem which is almost correct is marked all wrong; how does it affect you when each mathematics theory is presented as an absolute; what effect do the people who boast that they cannot understand mathematics have on you when these same people keep strangely silent about not understanding Shakespeare and Freud; what effect does the authoritarian attitude of mathematics teachers have on you? Why is the calculus easier for the child to understand than Shakespeare?

WILLIAMS, W. L. "What the Colleges Are Doing About the Poorly Prepared Student." *The American Mathematical Monthly*, February 1954, pp. 86-88.

We have all heard a great deal about the poor mathematics preparation of the students who enter college mathematics. Mr. Williams has made an extensive study to determine what the colleges are doing to remove the deficiency. You may not wholly agree with the implications but if the colleges lack confidence in the results of our teaching of high school mathematics, if the students do not perform up to the standards set by the colleges, then we need to come to a meeting of the minds. When the mathematics departments feel that one-half of the students we send them are poorly prepared, some discussion is called for.

This same study shows how some of the colleges are trying to meet the problem and also how some are ignoring it. I think you will want to read this article and consider what each of us is doing or can do so the college and secondary mathematics teachers can set a common level of proficiency and work together for its attainment.

KINZER, JOHN R. and LYDIA GREENE KINZER. "Some Bases for Predicting Marks in Advanced Engineering Mathematics," *Education Research Bulletin*, January 13, 1954.

This article contains valuable information for those of us who counsel our students as well as teach them mathematics. Did you know, for example, that most engineering students feel that mathematics is the most difficult part of their course? Do you think that courses in college can be used as a satisfactory substitute for courses the student could have taken in high school? Who has the best knowledge of mathematics, the major or the advanced engineering student? What types of mathematics do these advanced students need? Does the level of work in mathematics provide a good basis for predicting success? These are all questions whose answers are needed if we are to wisely counsel. The conclusion of this research will help.

MACDUFFEE, C. C. "So You Want to Be a Graduate Student." *Pi Mu Epsilon Journal*, November 1952, pp. 259-264.

Dr. MacDuffee has ably answered the statement that "a graduate student is a moron wandering around the campus not knowing the show is over." We who deal with the future Ph.D.'s in mathematics are in a crucial position to help prepare them for what is coming. This article points out what the graduates should have as a mathematics background, how devoted their interest needs to be, their need for initiative, and what type of past achievement usually leads toward success. He also gives some hints as: how to get started and what financial helps are available, what foreign languages to study and why, what particular type of course work should be pursued during the beginning graduate work. He helps one prepare himself for the environment, the competition and the possible future. This article will help us all in guiding our mathematics students who wish to continue in the field.—*Philip Peak, Indiana University, Bloomington, Indiana.*

## • WHAT IS GOING ON IN YOUR SCHOOL?

*Edited by John A. Brown, Wisconsin High School,  
University of Wisconsin, Madison 6, Wisconsin,  
and Houston T. Karnes, Louisiana State University, Baton Rouge 3, Louisiana*

### *Shall they take geometry?*<sup>1</sup>

*Contributed by Carmel Ling, Norwalk, California*

One hundred twenty-five letters were mailed to parents—and just two weeks before the end of the school year. Would *you* do this? We did, in the effort to create parent interest as well as to help counsel our students about taking geometry. This was only the last step in a long counseling program and the results we obtained paid for the effort involved.

The background of these letters is one with which all teachers are familiar. For the last five years the Mathematics Department at Excelsior High School has been making a real effort to counsel students planning to take geometry. Each year a new method has been tried; one year through social studies ("home room") teachers, another year through algebra teachers, another time through individual counseling by certain teachers in the Mathematics Department. Each method gives fine results at times, depending upon the individuals doing the counseling, but all the students did not receive the same guidance or information. As a result, many did not know whether to take geometry or not.

This year when the Lee Test of Geometric Aptitude was given, the algebra teachers giving it were asked to give each student a work-habit grade. Work-habit grade indicates the attitude the student takes toward his responsibility to his homework, class recitation, and the sub-

ject-matter. This is not a citizenship or achievement grade. This grade and his algebra aptitude score were written on the cover of the test booklet and sent to one teacher who graded all of the tests. After they were graded, a record card was made for each student including all this information and a previously determined arithmetic achievement score.

All of the mathematics teachers felt that one person should do all the counseling. When the cards were completed, each algebra teacher exchanged his class with this particular teacher. The teacher chosen to do this counseling was the one who had graded the tests, kept the records of the results for the last five years, and had made charts showing the relationship of these scores with the final geometry grades.

The teacher doing this counseling took these charts with her into each class and used them to show the students that their geometry grades would probably be between their work-habit grade and their aptitude score. The student was also shown that some students in the past have failed even with high aptitude scores while others have made average grades with low aptitude scores. She talked to the class as a group, answered their questions, and then called each student to the desk and showed him his own record card.

This was a rude awakening for some of the students, but each one took it well and said he was glad to know what his chances

<sup>1</sup> Reprinted by permission of the *The California Mathematics Council Bulletin* and taken from their November, 1953, issue.

were. No student was told that he could not take geometry, but he was advised against taking the subject if the evidence indicated his chances for success were small. Each student was then asked if he intended to take geometry and those who so indicated were asked to fill out a card with their parents' names and addresses. To each parent one of the following four letters was sent (the italics were not in the letters).

Dear ———:

At Excelsior we are making every effort we can to discover the abilities and interests of each student. In view of this practice we have given all of the algebra students geometry aptitude tests. The results of this test along with his work-habit grade in algebra seems to indicate that your son *should do very well in geometry if he continues as he has in the past.*

This information is offered so that you may better understand our guidance program in its effort to provide for the needs and abilities of our mathematics students at Excelsior High School. Should you care to discuss the results as it concerns your son and his plans for next year, our counselors will be happy to talk over the material with you at any time. They may be reached by telephone, Torrey 4-2721, and an appointment may be arranged for you if you so desire.

Sincerely,  
RICHARD GAHR, Principal

In the other three letters the italicized words were replaced by one of the following:

"should be able to carry geometry in a successful manner if he will apply himself diligently."

"will experience difficulty with geometry but should pass with continuous effort."

"will have great difficulty in understanding geometry and will have trouble passing the course."

For each of these four letters there is a similar one for the girls.

The letters were mailed Friday afternoon and Monday morning parents were calling the school and students were asking their algebra teacher what to do. Each parent who called was very pleased to get the letter and wanted to know how

he could help his son or daughter to do better in geometry. The parent wanted to know whether or not his son or daughter should take geometry, when the letter showed the student would have trouble with the subject. Some parents sent word that they appreciated the information and understood the implication.

Every parent contacting the school either directly or indirectly through his son or daughter was told that the letters were sent to help the student and his parents better understand the experiences he would have next year in geometry. The letter further advised the parents that if the student spent an adequate amount of time on the subject and still brought home an average or lower grade on his progress report, no one would feel that his time and effort had been wasted nor that he had done poorly. We also feel that we have made the way clear for grouping our students according to their subject aptitude, as many have requested.

It is the philosophy of our school that each student should be afforded an educational experience in which he can be successful, provided he works up to capacity; conversely, he should not elect a subject he cannot pass. We know that these letters will not keep all students who have little chance of passing geometry from taking it, but they will help prepare parents for the difficulties and for the poor grades some of their sons and daughters will have. There are always students in high school taking a college preparatory course who will never make the grade and are taking the work only to please their parents. These students should be counseled into taking other courses as early as possible; but unless the parents are advised as to their students' abilities and aptitudes little can be done. We feel that these letters can assist parents in helping their sons and daughters choose the correct courses. We in the Mathematics Department at Excelsior feel that we are making a real contribution to the individual guidance program.

# Reviews and evaluations

Edited by Richard D. Crumley, School of Education,  
University of South Carolina, Columbia, South Carolina,  
and Roderick C. McLennan, Arlington Heights High School, Arlington Heights, Illinois.

## BOOKS

*Emerging Practices in Mathematics Education*, Twenty-Second Yearbook, National Council of Teachers of Mathematics. Washington, D. C., National Council of Teachers of Mathematics, 1954. Cloth, v+434 pp., \$4.50, to members of the Council, \$3.50.

*Emerging Practices in Mathematics Education* is the Twenty-Second Yearbook of the National Council of Teachers of Mathematics. The yearbook is the combined product of its planning committee and the membership of the National Council. The committee published an announcement in the April 1952 issue of *THE MATHEMATICS TEACHER* stating its plan for the Twenty-Second Yearbook. Members of the Council responded to the article by proposing titles for the book, and volunteering contributions. The Planning Committee made its decision concerning the structure of the book upon consideration of suggestions proposed by its membership along with the suggestions received in response to the announcement. The committee agreed to organize the yearbook around the following categories of emerging practices: Part I, Various Provisions for Differentiated Mathematics Curriculum; Part II, Laboratory Teaching in Mathematics; Part III, Teacher Education; Part IV, New Emphases in Subject Matter; Part V, The Evaluation of Mathematical Learning; Part VI, A Bibliography of "What Is Going On In Your School?"

The reviewer was impressed with the recognition mathematics education leaders are giving to teaching practices that may more adequately educate mathematics students. Many of the articles advocate practices that will be appreciated by the teacher who wishes to make mathematics more meaningful. Teachers are challenged to make their classes both interesting and worth while, two objectives that require the presence of an alert teacher. The reviewer believes that teachers must be selective in adopting appropriate practices for their classrooms. Although each practice that is advocated may be excellent, all will not be found usable, for varied teaching objectives require differentiated practices.

Part V is appropriate in a book dealing with emerging practices, for it deals with the evaluation of mathematical learning. One is impressed with the pioneering done by those who wish to test both attitudes and appreciations. Teachers are searching for evaluation materials that will

assist them in determining students' awareness of concepts, separate from their learning of factual material.

Teachers will profit from the book in several ways. It furnishes evidence that some of the "new" practices of the past decade are becoming accepted practices today, information concerning some of the newest practices is presented, and support is given teachers experimenting with non-conventional practices.—Roderick C. McLennan, Arlington Heights High School, Arlington Heights, Illinois.

*Mathematics for Students of Engineering and Applied Science*, L. B. Benny. New York, Oxford University Press, 1954. Cloth, vi+783 pp., \$5.60.

The author of this excellent and very usable text is already well known for his previous texts: *Mathematics for Students of Technology, Junior Course, Parts I and II*, and *Senior Course, Parts I and II*, to which the present volume is a logical and admirable sequel.

This text is designed to meet, within the compass of one volume, the mathematical needs of students preparing for various engineering degrees, as well as students majoring in related fields of applied science and technology. In this reviewer's opinion, it meets these needs more than adequately, for it includes a wide range of topics in pure mathematics such as are today generally required of students in the fields mentioned. The author intimates that he had certain qualms about an "omnibus" book which would meet with general approval. He need not have had any concern; the selection of material is most judicious, from a technical as well as from a pedagogical viewpoint.

To be sure, the development is adapted to British secondary school curriculums, but this does not detract from its usefulness in American colleges and schools of engineering. The book begins, interestingly enough, with a discussion of convergence of series, based upon the notion of the limit of a sequence. These are important ideas, and they are well presented; a good balance is preserved between mathematical rigor and pedagogical realism. This first chapter includes the binomial theorem, exponential and logarithmic series, and hyperbolic functions. The next nine chapters (nearly 350 pages) deal, in order, with complex numbers, De Moivre's Theorem, algebraic equations, determinants, the elements of plane and solid coordinate geometry, vectors and vector products, differentiation

and applications, expansion of functions in series, integration and applications. This constitutes just about one-half of the book, and covers approximately what is dealt with in three or four semesters of "college algebra," analytic geometry, and a year of introductory calculus in the typical American college. There is this exception: the material has been selected and organized more effectively. For example, the coordinate geometry is happily limited to what is most likely to be needed by the engineer; the same is true of the algebraic theory.

The remaining nine chapters deal with differential equations, functions of more than one variable, partial differentiation, integration of linear differential equations in series, Bessel functions and Legendre functions, Fourier series and harmonic analysis, multiple integrals, Beta and Gamma functions, spherical trigonometry, differentiation of vectors, errors of observation, and the method of least squares.

This is obviously a thorough mathematical foundation for students of engineering and technology. It remains to point out a few remaining features of the book. The typography is remarkably clear, despite the regrettable use of small type. Also to be regretted is the absence of an index. However, there is an abundance of exercises, with answers to all that require no demonstration; the exercises appear to be fairly well graded. The style is simple and straightforward, and there are many illustrative examples worked out. There is no attempt at modern mathematical rigor; proofs of certain critical theorems have been frankly omitted, although, as the author quaintly puts it, he has tried to rise above the "pictures and plausibility" standard of the recent past. Naturally, when so much material is encompassed within the covers of one volume, its development is relatively rapid and rather succinct. All in all, the book is a welcome addition to the field.—*William L. Schaaf, Brooklyn College, Brooklyn, N. Y.*

## EQUIPMENT

*Burns Pupils' Boards (#856) and Work Sheets (#857)*, Ideal School Supply Company, 8312 Birkhoff Avenue, Chicago 20, Illinois. Ten laboratory boards, each  $9" \times 12"$ ; instruction sheet and student worksheet furnished with each board; \$5.00 for set of ten boards, or \$0.60 each; additional worksheets, \$0.25 per pad of 20 sheets.

These cardboard models are a smaller version of the *Burns Teachers' Boards* (#855) evaluated in May, 1954. Each of the ten boards is made of  $\frac{1}{4}"$  cardboard, punched with holes for pegs threaded with elastic cord. The cord then outlines a geometric figure. The purpose of the boards is to provide an opportunity for the student to discover inductively some of the properties of geometric figures. The ten boards and corresponding worksheets deal with: Triangles and Their Angles (P 1), Quadrilaterals and Parallelograms and Their Diagonals (P 2), Polygons (P

3), Pythagorean Theorem (P 4), Altitudes of Triangles (P 5), Circle Circumscribed about a Triangle (P 6), Triangles Equal in Area (P 7), Angles Inscribed in a Semi-circle (P 8), Medians of Triangles (P 9), and Perpendicular Bisector (P 10). The worksheets serve as a guide for forming the figures and, by the questions asked, help the student to discover the generalization.

In the opinion of the reviewer, the pupils' boards should be considered as basic instructional materials rather than as accessories to the teachers' boards. If every group of two or three students can work with a set of these boards and worksheets, much more discovery will take place than if there is only a set of the larger teachers' boards. The worksheets are well prepared, but this should not deter teachers from adapting or replacing them with some of their own.

These boards could be used effectively at the junior high school level in instruction in intuitive geometry or at the high-school level in the typical demonstrative geometry class. At the latter level, the boards would be used to discover some of the properties of geometric figures, after which the students could prove deductively that the property holds.

This device is admittedly rather expensive; but, since ten sets might well serve a class of thirty pupils, the cost would not be as great as one might think at first. The producer is to be commended for making the Burns boards available.—*Richard Crumley, University of South Carolina.*

*Construct-A-Globe. Models of Industry*, 2100 5th Street, Berkeley 2, California. Put-together model;  $10\frac{1}{2}"$  in diameter; \$2.95 each.

A sturdy cardboard box contains ten sections cut to go together to form a globe, wire pieces to form a supporting stand, a  $16"$  strip to measure distance on the globe's surface, a transparent, plastic map of the United States ( $4" \times 5"$ ) to slide around to compare areas, a page of thirty-two small, adhesive pictures to put on the surface of the globe; and a teacher's manual ( $8\frac{1}{2}" \times 11"$ , 28 pages). This manual contains directions for assembling the model and questions to stimulate its use.

This globe is fun to make, attractive to look at, instructive to use, and accurate to learn from. It is fairly substantial and would be more so if it were reinforced on the inside by Scotch Tape. The wire stand is rather feeble and would certainly give way eventually under normal, adolescent use. Many of the suggestions in the manual are excellent and would inspire mathematics students to learn about distance, direction, time zones, and latitude and longitude; but the total organization of the manual is confusing. It tries to do too many things, in too many ways, in too short a space. However, the inspiration of this well-designed kit for students to make other globes by similar techniques will certainly utilize the natural integration between the social studies and mathematics.—*Henry W. Syer, Boston University.*

## • MATHEMATICAL MISCELLANEA

*Edited by Paul C. Clifford, State Teachers College, Montclair, New Jersey,  
and Adrian Struyk, Clifton High School, Clifton, New Jersey.*

### "More-than-similar" triangles

*By Charles Salkind, Samuel J. Tilden High School, Brooklyn, New York*

On pages 295 and 296 of the April 1953 issue of *THE MATHEMATICS TEACHER*, Robert R. Halley of Avenal, California, writes of challenging his trigonometry students with this first assignment: "Bring to class two cardboard triangles, which have five parts of one triangle equal to five parts of the other, but the sixth parts unequal." He then develops the problem to the point of ruling out three sides since then the triangles would be congruent. Consequently, of the five equal parts—limiting ourselves to sides and angles only—three must be angles. The problem then, as he states it correctly, is to have two similar triangles with two of their three sides equal respectively, but not correspondingly placed.

Intrigued by the nature of the relation between the sides I followed up the matter thus:  
Let the triangles have sides  $a, b, c_1$  and  $a, b, c_2$  such that

$$\frac{a}{c_2} = \frac{b}{a} = \frac{c_1}{b}, \quad c_2 > c_1.$$

Therefore

$$(1) \quad b^2 = ac_1, \quad a^2 = bc_2.$$

From (1) it is plain that

$$b^2/a^2 = (a/b)(c_1/c_2),$$

and therefore

$$(2) \quad b/a = (c_1/c_2)^{1/3}$$

Then from (1) and (2) it follows that

$$(3) \quad a = (c_1 c_2^2)^{1/3}, \quad b = (c_1^2 c_2)^{1/3}.$$

Now

$$b + a > c_2$$

because of the triangle, and this with the relations in (3) yields

$$(c_1/c_2)^{2/3} + (c_1/c_2)^{1/3} > 1.$$

Let  $r = (c_1/c_2)^{1/3}$ . Then

$$r^2 + r > 1,$$

whence it follows that

$$r > \frac{1}{2}(-1 + \sqrt{5}).$$

Finally, since  $c_2 > c_1$  we may write

$$1 > r > \frac{1}{2}(-1 + \sqrt{5}).$$

It is interesting to note once again the unexpected appearance of the Fibonacci constant  $\frac{1}{2}(-1 + \sqrt{5})$ .

We observe that  $c_2, a, b, c_1$  are in geometric sequence with common ratio  $r$ . Since

$$.61 < (-1 + \sqrt{5}) < .62$$

a geometric sequence of four numbers taken with .61 as the common ratio fails to give us a constructible pair of triangles, while one with .62 as the common ratio does give us a constructible set. For example, with  $r = .61$  take  $c_2 = 100$ . Then  $c_2 = 100$ ,  $a = 61$ ,  $b = 37.21$ ; and  $a = 61$ ,  $b = 37.21$ ,  $c_1 = 22.6981$ . Here the sum of the

two shorter lengths of each set is less than the third length, and no triangles are possible. Now with  $r = .62$  take  $c_2 = 100$ . Then  $c_2 = 100$ ,  $a = 62$ ,  $b = 38.44$ ; and  $a = 62$ ,  $b = 38.44$ ,  $c_1 = 23.8328$ . Here triangles are possible, for the sum of the two smaller sides is just greater than the third side.

Of course, we can obtain a large number of satisfactory triangles, even if we limit

ourselves to integral values. With  $r = \frac{2}{3}$  take  $c_2 = 27$ . Then

$$c_2 = 27, \quad a = 18, \quad b = 12;$$

$$a = 18, \quad b = 12, \quad c_1 = 8.$$

With  $r = \frac{3}{4}$  take  $c_2 = 64$ . Then

$$c_2 = 64, \quad a = 48, \quad b = 36;$$

$$a = 48, \quad b = 36, \quad c_1 = 27.$$

## What's new?

### BOOKS

#### SECONDARY

*Algebra: Its Big Ideas and Basic Skills, Book 1*, 1954 edition, Daymond J. Aiken, Kenneth B. Henderson, New York, McGraw-Hill Book Company, Inc., 1954. Cloth, iii+419 pp., \$2.84.

*Algebra: Its Big Ideas and Basic Skills, Book 2*, Daymond J. Aiken, Kenneth B. Henderson, New York, McGraw-Hill Book Company, Inc., 1954. Cloth, v+397 pp., \$3.96.

*Mathematical Puzzles and Pastimes*, Aaron Bakst, New York, D. Van Nostrand Company, Inc., 1954. Cloth, vi+206 pp., \$3.75.

#### COLLEGE

*Diagrams in Punched Card Computing*, Fred Gruenberger, Madison, University of Wisconsin Press, 1954. Paper, vii+108 pp., \$3.75.

*Elements of Number Theory*, I. M. Vinogradov, translated from the Fifth Revised Edition by Saul Kravetz, New York, Dover Publications, Inc., 1954. Cloth, iii+227 pp., \$3.00, paper, \$1.75.

*Limit Distribution for Sums of Independent Random Variables*, Gnedenko and Kolmogorov, Cambridge, Addison-Wesley Publishing Company, Inc., 1954. Cloth, ix+234 pp., \$7.50.

*Noise*, Aldert van der Ziel, New York, Prentice-Hall, Inc., 1954. Cloth, vii+450 pp., \$7.75.

#### MISCELLANEOUS

*Perceptualistic Theory of Knowledge*, Peter Fireman, Ph.D., New York, Philosophical Library, Inc., 1954. Cloth, ii+50 pp., \$2.75.

*Rotating Electrical Machinery*, Universal Scientific Company, Vincennes, Indiana, Crow Electric-Craft Corporation, 1954. Paper, 256 pp., \$3.50.

*Stone Age Economists in the Atomic Age*, Olaf Nelson, New York, Pageant Press, 1954. Cloth, v+160 pp., \$3.00.

*The Geometry of René Descartes*, translated from the French and Latin by David Eugene Smith and Marcia L. Latham, New York, Dover Publications, 1954. Cloth, vi+243 pp., \$2.95, paper, \$1.50.

### BOOKLETS

*A Slide Rule with Intellect and Horse Sense*, Keuffel & Esser Company, Adams and Third Streets

Hoboken, New Jersey  
Booklet in color, 14 pp., free

*Your Opportunities in Science and Engineering*  
Education Department, National Association of Manufacturers  
14 West 49th Street, New York 20, New York  
Booklet, 32 pp., free

### EQUIPMENT

*Numeraid*  
Peerless Manufacturing and Distributing Company  
Waukesha, Wisconsin  
American abacus with five rods, \$2.00

### FILMS

*How to Use Consumer Credit Wisely*  
Consumer Education Department, Household Finance Corporation  
919 North Michigan Avenue, Chicago 11, Illinois

B & W, silent, 35 mm. filmstrip, accompanied by script; free except for return postage

*Piercing the Unknown*  
Modern Talking Picture Service, Inc.  
45 Rockefeller Plaza, New York 20, New York

16 mm. sound film in color dealing with electronic calculating, 22 min., produced by IBM Corp., free except for postage and insurance

# THE MATHEMATICS TEACHER

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*Henry Van Engen, Iowa State Teachers College, Cedar Falls, Iowa*

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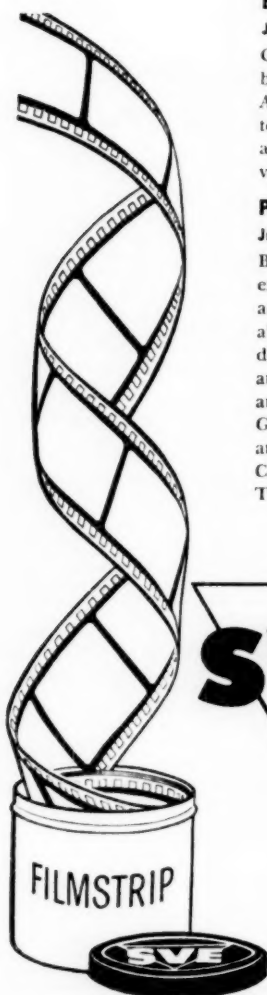
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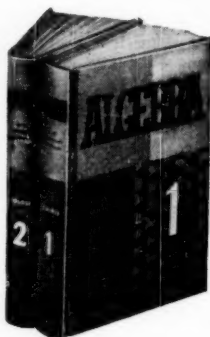
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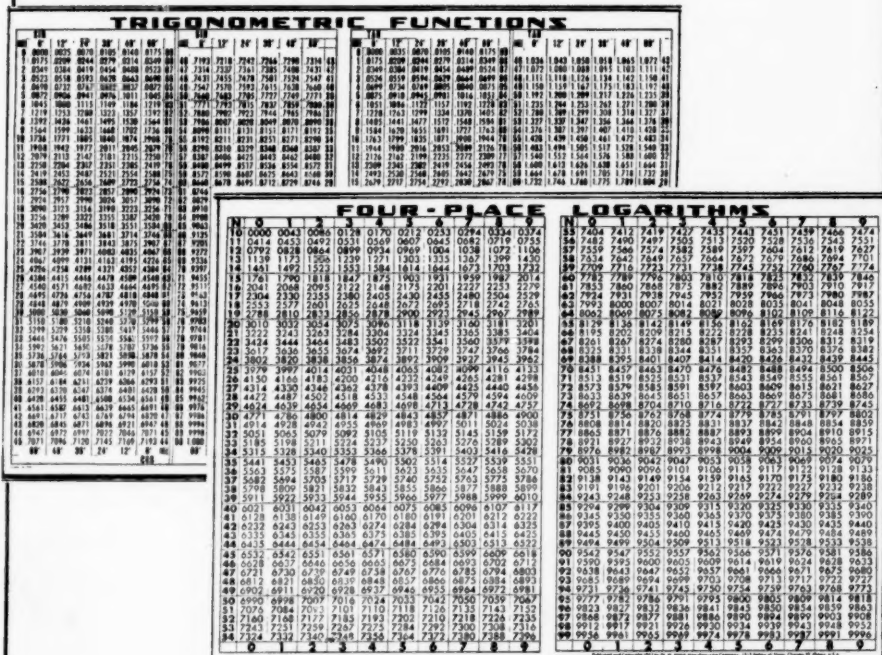
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